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TOPICAL REVIEW

A review of the recent research on vibration energy harvesting via bistable systems

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Abstract
The investigation of the conversion of vibrational energy into electrical power has become a major field of research. In recent years, bistable energy harvesting devices have attracted significant attention due to some of their unique features. Through a snap-through action, bistable systems transition from one stable state to the other, which could cause large amplitude motion and dramatically increase power generation. Due to their nonlinear characteristics, such devices may be effective across a broad-frequency bandwidth. Consequently, a rapid engagement of research has been undertaken to understand bistable electromechanical dynamics and to utilize the insight for the development of improved designs. This paper reviews, consolidates, and reports on the major efforts and findings documented in the literature. A common analytical framework for bistable electromechanical dynamics is presented, the principal results are provided, the wide variety of bistable energy harvesters are described, and some remaining challenges and proposed solutions are summarized.

(Some figures may appear in colour only in the online journal)

1. Introduction

Vibrational energy harvesting studies have begun adopting the perspective that linear assumptions and stationary excitation characteristics used in earlier analyses and designs are insufficient for the application of harvesters in many realistic environments. The principal challenge is that linear oscillators, well suited for stationary and narrowband excitation near their natural frequencies, are less efficient when the ambient vibrational energy is distributed over a wide spectrum, may change in spectral density over time, and is dominant at very low frequencies [1, 2].

These factors encouraged the exploration of methods to broaden the usable bandwidth of linear harvesters, including oscillator arrays, multi-modal oscillators, and active or adaptive frequency-tuning methods [3, 4]. While providing improvements, more advanced solutions were desired for broadband performance, and the exploitation of nonlinearity became a subsequent focus. To date, a number of nonlinear energy harvesting studies have been conducted, mostly focusing on the monostable Duffing [5–7], impact [8, 9], and bistable oscillator designs. Monostable Duffing harvesters exhibit a broadening resonance effect dependent on the nonlinearity strength, device damping, and excitation amplitude, and thus can widen the usable bandwidth of effective operation. Impact harvesters provide a mechanism for frequency up-conversion by using lower ambient vibration frequencies to impulsively excite otherwise linear harvesters so that they may ring down from much higher natural frequencies.

Bistable oscillators have a unique double-well restoring force potential, as depicted in figure 1. This provides for three

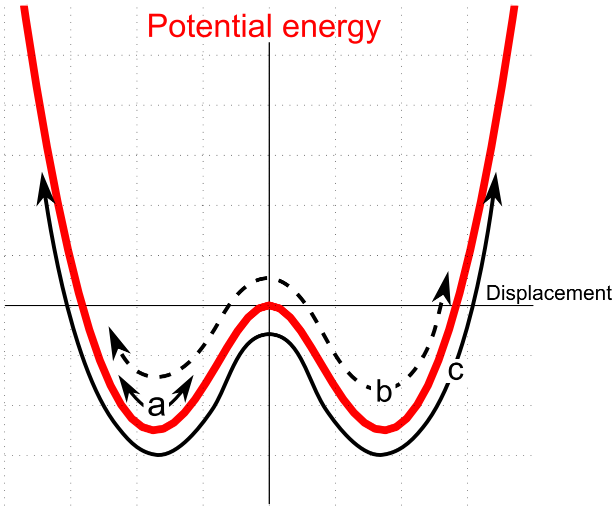

Figure 1. Double-well restoring force potential of a bistable oscillator showing example trajectories for (a) intrawell oscillations, (b) chaotic interwell vibrations and (c) interwell oscillations.

distinct dynamic operating regimes depending on the input amplitude, figure 2. Bistable devices may exhibit low-energy intrawell vibrations (figure 1(a)). In this case, the inertial mass oscillates around one of the stable equilibria with a small stroke per forcing period; see the example displacement–time response trajectory (figure 2(a)) and phase portrait with an overlay Poincare map (figure 2(d)). Alternatively, the bistable oscillator may be excited to a degree so as to exhibit aperiodic or chaotic vibrations between wells (figures 1(b), 2(b) and (e)). As the excitation amplitude is increased still further, the device may exhibit periodic interwell oscillations (figures 1(c), 2(c) and (f)). In some cases, the dynamic regimes may theoretically coexist although only one is physically realizable at a time.

The periodic interwell vibrations—alternatively, high-energy orbits or snap-through—have been recognized as a means by which to dramatically improve energy harvesting performance [3, 4]. As the inertial mass must displace a greater distance from one stable state to the next, the requisite velocity of the mass is much greater than that for intrawell or chaotic vibrations. Since the electrical output of an energy harvester is dependent on the mass velocity, high-energy orbits substantially increase power per forcing cycle (as compared with intrawell and chaotic oscillations) and are more regular in waveform (as compared with chaotic oscillations), which is preferable for external power storage circuits. Additionally, snap-through may be triggered regardless of the form or frequency of exciting vibration, alleviating concerns about harvesting performance in many realistic vibratory environments dominated by effectively low-pass filtered excitation [10].

These benefits have instigated a rapidly growing body of literature on bistable energy harvesting. Among many, three common bistable harvester concepts are depicted in figure 3. Harvesting circuitry is indicated by the parallelogram, and attached piezoelectric patches for converting mechanical strain to electrical energy are shown as light gray layers partially covering the beam lengths. The direction of base excitation is indicated by the double arrows. Figure 3(a) shows a magnetic repulsion harvester with the strength of the nonlinearity governed by the magnet gap distance d_r. Figure 3(b) shows a magnetic attraction bistable harvester using a ferromagnetic beam directed towards one of two magnets separated a distance $2d_g$ from each other and d_a from the end of the beam. Lastly, figure 3(c) shows an example of

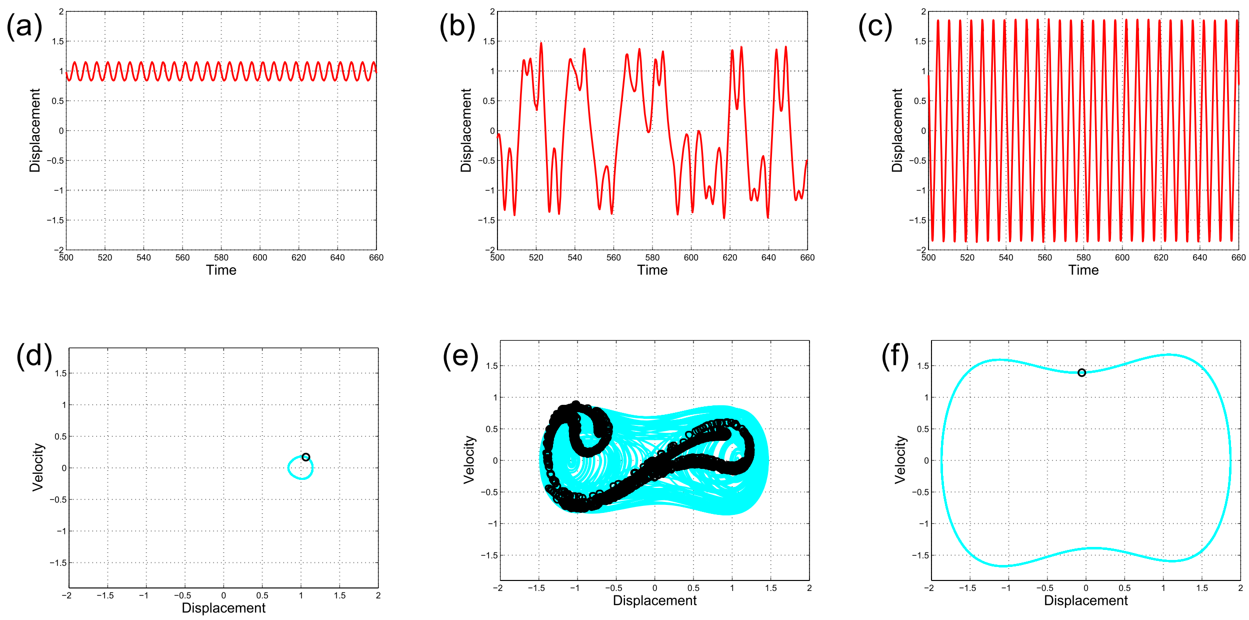

Figure 2. Example displacement–time responses (top row) and phase plots with an overlap Poincare map as black circles (bottom row) for three dynamic regimes of bistable oscillators. (a) and (d) Intrawell oscillations. (b) and (e) Chaotic vibrations. (c) and (f) Interwell oscillations.

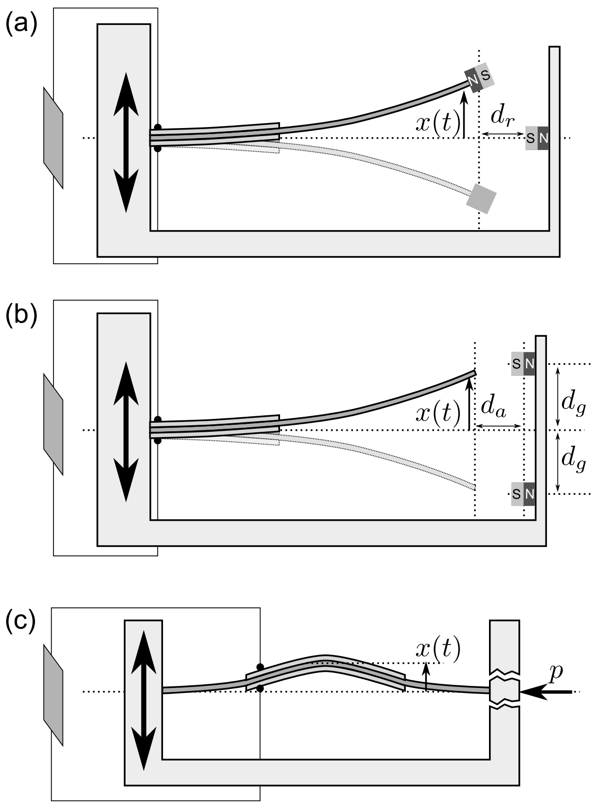

Figure 3. (a) Bistable magnetic repulsion harvester. (b) Magnetic attraction harvester. (c) Buckled beam harvester. Piezoelectric patches shown as light gray layers along part of the beam lengths. Harvesting circuitry shown as parallelograms.

a buckled beam harvester with the bistability modified by a variable axial load p. Note that while these three are used here as examples for illustration, this review is not limited to these types of devices.

The fundamental electromechanical dynamics have been evaluated analytically and experimentally with respect to the individual bistable device design under consideration, but a common dimensionless formulation is often utilized that yields trends comparable across platforms. The advantages of bistability in both stationary and stochastic vibratory environments have also been detailed. Numerous studies have probed these subjects to various levels of refinement. As a result, a rigorous and comprehensive review of the bistable energy harvesting literature would be an important service to the technical community. A previous paper has summarized a portion of bistable energy harvesting developments to date, though the authors impart particular emphasis to survey studies regarding MEMS-scale utility and efficiency metrics [11]. In contrast to the prior survey, the objective of the present review is to provide a comprehensive outline of the recent bistable energy harvesting literature, so as to encompass the breadth of work accomplished and provide sufficient attention to critical results of these studies.

In the following sections, this review organizes the variety of research investigations in bistable energy harvesting based on similar analytical methods and experimental conceptions. Following the presentation of a unified electromechanical analytical model widely employed by researchers, principal conclusions from analytical studies are summarized. Thereafter, the great body of experimental work is surveyed and additional insights observed experimentally but not captured in fundamental analysis are highlighted. Finally, remaining challenges to the field, proposed solutions to these obstacles, and the relation between bistable energy harvesting and similar explorations in contemporaneous fields are summarized.

2. Governing equations of single degree-of-freedom bistable oscillator

The interest in bistable oscillator dynamics grew in proportion to the discovery of the attendant chaotic oscillations which occur for specific operating parameters, first observed numerically and experimentally by Tseng and Dugundji [12]. The authors described the chaotic vibrations of a buckled beam as 'intermittent snap-through' [12]. Extensive exploration was performed later by Holmes [13] and Moon and Holmes [14] so that a more detailed understanding developed from which the recent literature in bistable energy harvesting has taken root. The governing equation derived was for a mechanically buckled beam [13] and for a beam buckled via magnetic attraction [14]. Using a one-mode Galerkin approximation, the authors derived an ordinary differential governing equation for the buckled beams which was found to accurately represent experimental results.

The governing equation for an underdamped, single degree-of-freedom oscillator excited by base acceleration may be formulated from the physical coordinates where the relative displacement $X(t)$ of an inertial mass m is determined by

$$m\ddot{X} + c\dot{X} + \frac{\mathrm{d}U(X)}{\mathrm{d}X} = -m\ddot{Z} \tag{1}$$

where c is the viscous damping constant, $\ddot{Z}$ is the input base acceleration, and the overdot denotes differentiation with time. The restoring force potential of the oscillator may be expressed as

$$U(X) = \tfrac{1}{2}k_1(1-r)X^2 + \tfrac{1}{4}k_3X^4 \tag{2}$$

where k_1 is the linear spring constant, k_3 is the nonlinear spring constant, and r is a tuning parameter. Figure 4 shows the effect on the restoring force potential for three cases of tuning parameter and nonlinearity strength, $\delta = k_3/k_1$. The linear oscillator, $\delta = 0$ and $r < 1$, is monostable as is the nonlinear Duffing oscillator, $\delta \neq 0$ and $r \leq 1$ which exhibits a softening nonlinearity for $\delta < 0$ and hardening nonlinearity when $\delta > 0$. However, when the tuning parameter $r > 1$ and $\delta > 0$, the central equilibrium is no longer stable and the system becomes nonlinear bistable, having new stable equilibria at $X^* = \pm\sqrt{(r-1)/\delta}$. This latter case is also referred to as the Duffing–Holmes oscillator in honor of their collective contributions [15].

A nondimensional time, $\tau = \omega t$, is applied to equation (2) where $\omega = \sqrt{k_1/m}$ is the linear natural frequency of the oscillator. Defining $\zeta = c/2m\omega$, and operator $(\cdot)'$ as differentiation with respect to τ, the nondimensional

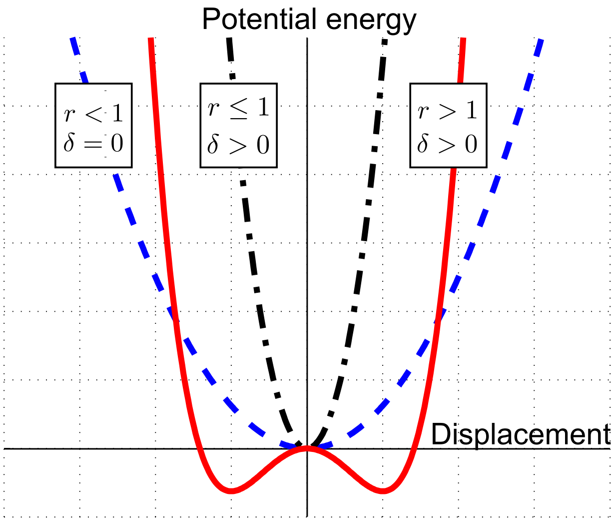

Figure 4. Spring force potential as tuning and nonlinearity are modified.

governing equation is given as

$$x'' + 2\zeta x' + (1 - r)x + \delta x^3 = -z''. \tag{3}$$

Equation (3) is the Duffing equation, which has a rich history [15]. Despite representing a purely mechanical system, equation (3) served as the initial model for a number of bistable energy harvesting studies [16–18]. This convention stems from early energy harvesting literature in which the coupled external circuit was modeled as equivalent damping [19]. While this is an incomplete perspective, it provides fundamental insight into the role of bistability in electromechanical dynamics and justifies its adoption.

On the other hand, coupling effects between the external harvesting circuit and the electromechanical device have been rigorously studied and verified, particularly as related to harvesting efficiency [20–23]. As a result, more recent studies have included a coupled external circuit equation for greater fidelity. Depending on both the manifestation of bistability that is considered and the form of electromechanical coupling, different equations of motion in physical coordinates are attained. Drawing on a number of recent works [24–27], the authors here provide a framework which collects together the electromechanical conversion mechanisms most often considered: electromagnetic and piezoelectric.

Figure 5(a) shows a generic electromechanical oscillator with restoring force $\mathrm{d}U/\mathrm{d}x$, base excitation $\ddot{Z}$, and piezoelectric and electromagnetic conversion mechanisms. The subsequent external circuits connected to the piezoelectric and electromagnetic mechanisms are depicted in figures 5(b) and (c), respectively. To adopt a common convention in the literature, the external harvesting circuits are described by a generic load resistance [24–27]. The governing equations are found to be

$$m\ddot{X} + c\dot{X} + \frac{\mathrm{d}U(X)}{\mathrm{d}X} + \theta V + \gamma I = -m\ddot{Z} \tag{4}$$

$$C_{\mathrm{p}}\dot{V} + \frac{1}{R_1}V - \theta\dot{X} = 0 \tag{5}$$

$$L\dot{I} + R_2 I - \gamma\dot{X} = 0 \tag{6}$$

where θ is the linear piezoelectric coupling coefficient; V is the voltage across the load resistance R_1 for the piezoelectric harvesting component; C_{p} is the capacitance of the piezoelectric material; γ is the electromagnetic coupling coefficient; I is the current through the load resistance R_2 for the electromagnetic harvesting component, where the total resistance is the sum of a coil resistance and the harvesting circuit resistance; and L is the inductance of the electromagnetic mechanism.

Introducing new coordinates

$$\begin{aligned} x &= X; & z &= Z; \\ v &= C_{\mathrm{p}}V/\theta; & i &= LI/\gamma \end{aligned} \tag{7}$$

and employing the nondimensional time, $\tau = \omega t$, where $\omega = \sqrt{k_1/m}$ is again the linear natural frequency of the oscillator, the dimensionless system of equations is determined as

$$x'' + 2\zeta x' + (1 - r)x + \delta x^3 + \kappa^2 v + \mu^2 i = -z'' \tag{8}$$

$$v' + \alpha v - x' = 0 \tag{9}$$

$$i' + \beta i - x' = 0 \tag{10}$$

where the following variables are defined

$$\begin{aligned} 2\zeta &= \frac{c}{m\omega}; & \delta &= \frac{k_3}{k_1}; & \kappa^2 &= \frac{\theta^2}{k_1 C_{\mathrm{p}}}; \\ \mu^2 &= \frac{\gamma^2}{k_1 L}; & \alpha &= \frac{1}{R_1 C_{\mathrm{p}}\omega}; & \beta &= \frac{R_2}{\omega L}. \end{aligned} \tag{11}$$

In this notation, κ and μ are linear piezoelectric and electromagnetic coupling coefficients, respectively; while α and β are the nondimensional frequencies of the piezoelectric and electromagnetic components, respectively, normalized relative to the linear natural frequency of the mechanical oscillator. Bistable energy harvesting studies focus on the case in which the tuning parameter $r > 1$. Furthermore, several works cited in this review define the negative linear stiffness such that $r = 2$, reducing the number of parameters in equation (8).

3. Analysis approaches and results

Equations (8)–(10) are the foundation for many recent bistable energy harvesting analyses. Should an individual study be concerned with only one of the electromechanical conversion methods, the unrelated equation and coupling components are omitted. Although not all authors report their exact approach, a variety of analytical techniques exist to predict the response of a bistable system governed by equation (8)–(10). The type of information produced by each method is unique and the principal insight obtained is characteristic to the analytics; thus, it is natural to distinguish the key results based on the analytical methods.

3.1. Numerical integration

Following conversion to state-space, many studies thereafter predict system response via numerical integration. The

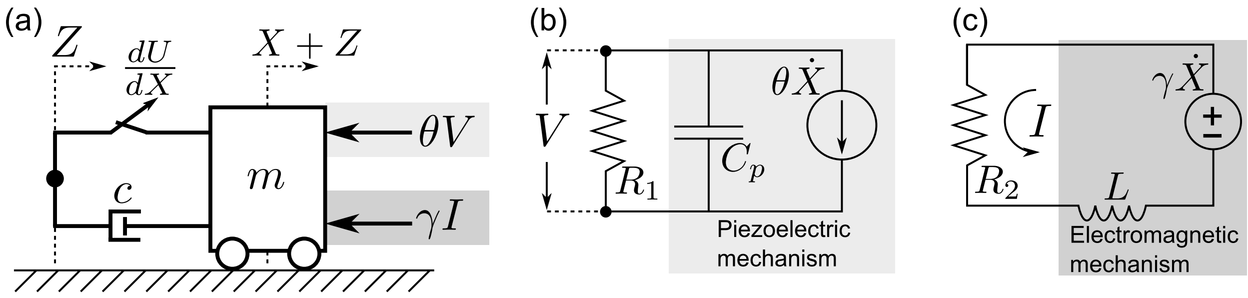

Figure 5. (a) Representative mechanical schematic of bistable energy harvester having both piezoelectric and electromagnetic conversions. (b) Equivalent coupled circuit for piezoelectric element. (c) Equivalent circuit for electromagnetic element.

benefits to this approach are the relative straightforward nature of simulation and the versatility to model arbitrary input excitations. The main drawbacks are the computational time and cost for the requisite repeated simulations if one is interested in frequency-domain information and the difficulty in attaining a deep and comprehensive insight into the system dynamics.

Erturk and Inman [26] used numerical integration to compare the qualitative similarity of simulated and measured data for a bistable piezoelectric harvester. The authors found good agreement for time-domain predictions in terms of the bistable harvester outperforming the linear equivalent over a broad range of frequencies. In the study, the linear device and the bistable harvester exhibited similar linearized natural frequencies to provide a meaningful comparison of performance. Apart from near the linear harvester's natural frequency, the bistable harvester consistently yielded greater levels of output power, so long as the bistable device maintained a high-energy orbit. In the event that chaotic oscillations were induced, the bistable device provided only a marginal increase in RMS voltage output.

Several studies have predicted system response to random noise inputs via numerical integration [28–30]. Litak *et al* [29] observed that a certain level of white Gaussian input appeared to maximize the output power of the device, which was explained to be the result of inducing a form of stochastic resonance. This phenomenon is the combined result of a small periodic force acting on the oscillator, so as to create a dynamic double-well potential, and a certain level of input noises which collectively induce dramatic interwell oscillations [31, 32]. McInnes *et al* [17] proposed exploiting this feature in energy harvesting, providing simulations in which the triggering of stochastic resonance significantly improved the bistable harvester performance compared to intrawell vibrations. The study by Litak *et al* [29] did not provide for a dynamic double-well potential but did observe that an optimum level of stochastic excitation existed even for a static restoring force potential to maximize output voltage.

A bistable plate having piezoelectric patches for energy harvesting has been investigated [33, 34]. The system response was assumed to be the coupled dynamics of the two unique stable modes and a subharmonic behavior. Rather than a continuum approach, the three coupled responses were approximated as individual out-of-plane displacements at a given point on the plate, thus representing the relative contribution of the three responses to a given excitation. Curve fitting with a quadratic polynomial was used to characterize the nonlinear restoring force. In spite of the simplifying assumptions, the numerically integrated simulations agreed well with experimental data, particularly given the discontinuous nonlinearity of the restoring force [35, 36].

3.2. Harmonic balance

Harmonic balance is advantageous for providing an efficient analytical framework to assess steady-state dynamics. The drawbacks are inherent in the assumption that the system response is the superposition of a number of harmonics, and therefore the fidelity of the method is limited to the size of the truncated series.

Stanton *et al* [37] found that the optimum electromechanical coupling strength for energy harvesting was the maximum value that sustained high-energy orbits. The analysis was also used to predict optimal load impedance conditions for energy harvesting from snap-through vibrations; this characteristic was demonstrated experimentally as well [26]. The results were compared against direct numerical integration and showed good agreement [37].

Mann *et al* [38] used harmonic balance to evaluate the effect of uncertainties inherent in a realistic energy harvesting application, e.g., device design parameters or excitation characteristics. A bistable harvester was compared against linear, softening monostable Duffing, and hardening monostable Duffing designs. Although the linear device was predicted to provide greater average output power than all of the nonlinear devices at the linear resonance frequency, the 95% confidence interval for the linear device was substantial, suggesting a high susceptibility of performance to parameter changes. In contrast, the bistable harvester exhibited the most consistent performance, having tightly confined confidence intervals around the average. This provides proof of the robustness of the bistable harvester to a changing excitation environment as well as imperfect knowledge of design parameters.

3.3. Method of multiple scales (MMS)

The method of multiple scales yields steady-state and transient solutions under the assumption of small perturbation of vibrations. Karami and Inman [25] used MMS in the analysis of the bistable energy harvester with an expansion to three time scales. It was shown that the MMS solution had reduced efficacy as the vibration amplitude around an equilibrium state was increased. The successful use of MMS was verified for small deviations around either the intrawell and interwell vibration solutions. The method was also utilized to determine an equivalent damping and frequency shift induced by energy harvesting so as to reduce analysis to a single governing equation. Simulations using this approach agreed well with results computed from the fully coupled electromechanical equations. Since a number of works in bistable energy harvesting utilize only the mechanical governing equation (3) to approximate energy dissipated, the equivalency provides a nontrivial and computationally efficient correction to such analyses.

3.4. Melnikov's method

The determination of design or excitation parameters necessary to maintain high-energy orbits is crucial to the optimum design of a bistable energy harvester. To this end, Melnikov's theory may be employed. Melnikov's method stems from the study of conditions suitable for homoclinic bifurcations which characterize the transition of a bistable system's dynamics into the chaotic regime [15, 39, 40]. The key disadvantage to the approach is its conservative estimation of the onset of bifurcation, thus limiting its usefulness as a design tool.

Following derivation of the mechanical governing equation, the seminal work of Holmes [13] employed Melnikov's theory to determine the critical nondimensional amplitude for single-frequency sinusoidal excitation. As this was a purely mechanical formulation, this represents the open-circuit result for the bistable energy harvester.

Stanton *et al* [41] applied Melnikov's method to study the bistable piezoelectric energy harvester. For single-frequency excitation, it was shown that the onset of homoclinic bifurcation was most sensitive when the driving frequency was 0.765ω, and that sensitivity was not a function of system damping. A normalized load impedance was determined to yield the greatest electrical damping for the harvester, thereby inhibiting interwell vibrations. The inclusion of electromechanical coupling to Melnikov's analysis of the bistable oscillator was predicted to be so influential as to be capable of destabilizing interwell oscillations. The authors also demonstrated that arbitrary multi-frequency excitation could be exploited via the theory to induce interwell oscillations, when the individual single-frequency excitations were insufficient for that purpose. For white Gaussian excitation, the bistable energy harvester was predicted to provide approximately equivalent levels of power as compared to the linear harvester design. For exponentially correlated noise excitation, a dramatic advantage of the bistable harvester was demonstrated. Despite the conservative estimates predicted by the Melnikov method, the study provided new insights into the role of the electromechanical coupling in inducing (or repressing) interwell vibrations [41].

3.5. Stochastic differential equation solution

Several works have documented the advantage of bistable energy harvesters over their linear counterparts when the excitations are stochastic [10, 27, 42, 43]. Daqaq [10] studied the solution to the Fokker–Planck–Kolmogorov (FPK) equations in the event of white Gaussian and exponentially correlated noise input. It was found that when excited by white Gaussian noise, linear and bistable inductive harvesters yield the same mean output power, a conclusion also verified via Melnikov's method [41]. However, Daqaq [10] noted that many real-world stochastic vibration sources are not purely white and would be more accurately represented as exponentially correlated noise. In such cases, it was demonstrated how the double-well potential may be designed so as to yield greater power output from the bistable harvester than the linear device. Furthermore, optimal double-well potential shapes could be determined for inducing interwell vibrations for exponentially correlated noise excitation. It was shown that such potential shapes led to corresponding optimum escapement frequencies, similar in effect to the Kramer's rate in the study of stochastic resonance [31]. These results were also verified by corresponding numerically integrated simulations [10].

Andò *et al* [42] and Ferrari *et al* [43] utilized the SDE Toolbox for MATLAB [44] to simulate the mechanical response of bistable harvesters to white Gaussian noise. Both studies observed that the power spectral density (PSD) of the bistable mass velocity was greater than that for the linear sample, except at the linear device natural frequency. These results along with the work of Daqaq [10, 27] demonstrate the robustness of bistable energy harvesters in stochastic vibration environments.

3.6. Signal decomposition

When a bistable oscillator is excited at frequencies much less than the linear natural frequency, the resulting displacement trajectory may exhibit a combination of slow and fast time scale oscillations, figure 6. Thus, the slow excitation forces the oscillator to jump across the double-well potential, where it rings down at its linear natural frequency before the input excitation forces it back across the double well: a frequency up-conversion technique. Cohen *et al* [45] used slow–fast decomposition to assess the dynamics of the system for this type of excitation. The approach was shown to accurately predict the force applied to a bistable harvester to induce interwell escape and thereafter serve as an impulsive excitation. Comparable experiments were carried out to validate the analytical approach. The decomposition technique was shown to provide a means by which to characterize the effectiveness in frequency up-conversion for the bistable harvester and serves as a tool for design optimization. In addition, the authors adapted the double-well potential used for simulation so as to be asymmetric for better comparison with experimental data. This represents one of the few attempts in literature to date to address asymmetry in the bistable harvester restoring force potential.

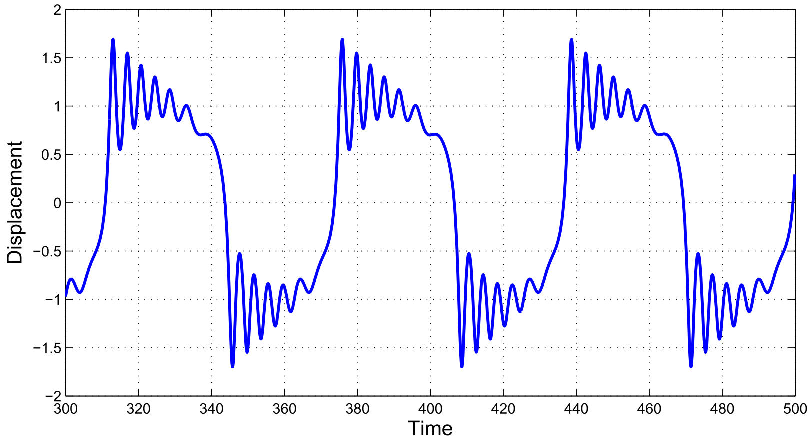

Figure 6. Steady-state solution, computed by numerical integration, of equation (3) for $-z'' = f\cos(\omega^*\tau)$, where ω^* is the ratio of the driving frequency to the linear natural frequency ω. Parameters: $r = 2$, $\delta = 1$, $\zeta = 0.1$, $f = 0.5$, $\omega^* = 0.1$.

4. Bistable device designs and experimental studies

The above analytical investigations employed similar mathematical models, generally of the form as presented in equation (8)–(10). However, the intended physical embodiment for each application varied from study to study. This section summarizes the various manifestations of bistable energy harvesters that have been designed and experimentally investigated, categorized by their bistability mechanisms.

4.1. Magnetic attraction bistability

The use of magnetic attraction to induce the bistability of a cantilevered ferromagnetic beam was one of the first investigations employed in studying the aperiodic chaotic response of an otherwise deterministic mechanical system [14]. It was this construction with an additional piezoelectric patch for energy harvesting, as in figure 3(b), which was explored by Erturk and Inman [26] for single-frequency excitation. An order of magnitude increase in power was generated for the bistable device, except at the linear harvester natural frequency, in which case the comparable linear device provided greater power. Chaotic oscillations were not sufficient to yield substantially greater RMS voltage than the linear device. Exceptional agreement was also observed between simulated and measured strange attractors of the bistable harvester [46].

Galchev *et al* [47] exploited impulsive snap-through as a frequency up-conversion technique to excite linear harvesting devices. In their configuration, a centrally suspended magnet is attracted by two end-suspended magnets along the axis of a tube. As base excitation increases, the central magnet is attracted so as to magnetically attach to one of the end magnets. The continued sinusoidal excitation thereafter causes the release of the central magnet from one end, which allows the end magnet to ring down through the axis of a coil, thus inducing flow of current in a harvesting circuit. The central magnet then snaps over and connects to the opposite end magnet. Upon the magnets releasing due to another half-cycle of input vibration, the end magnet rings down through a coil, while the central magnet snaps back to the opposite end magnet and the cycle repeats. Since the dynamics of operation are more affected by the input excitation amplitude than by the frequency, a unique measure of efficiency was proposed to show the advantage of the device in achieving broadband energy harvesting [47].

4.2. Magnetic repulsion bistability

There are numerous studies that have investigated bistable energy harvesters using magnetic repulsion to destabilize the linear equilibrium position [48–54]. Several of these investigations have considered a cantilevered piezoelectric beam with magnetic tip mass, having the same polarity as a facing magnet which may be moved a certain distance to the beam end so as to tailor the strength of the bistability (figure 3(a)). One feature of this configuration is that the repulsive magnets may be moved a great distance away so as to remove the nonlinearity and provide for the comparison against an equivalent linear harvester.

Lin and Alphenaar [50] showed that the bistable harvester design of figure 3(a) consistently yielded greater peak voltage than the equivalent linear device when excited by pink noise. The study utilized a rectifying circuit to compare the voltage measured on a storage capacitor. It was observed that the bistable harvester provided 50% greater voltage than the linear device.

Tang *et al* [53] also studied the bistable piezoelectric beam with magnetic repulsion. An optimum magnetic repulsion gap was observed, at which a considerable increase in broadband power could be harvested. The voltage in a storage capacitor was also approximately 50% greater than that of the linear harvester when the systems were excited by low-pass filtered stochastic vibration.

In a different experimental configuration than the prior, Tang *et al* [53] used repulsive magnets to induce a ring-down behavior from low input frequencies representative of wave heaves. In this investigation, the piezoelectric beam having a magnet tip mass remains stationary while a repulsive magnet (connected to the slow-heaving vibration source) passes near the beam tip, thus destabilizing the equilibrium position of the beam and causing it to vibrate as if struck by an impulse. It was observed that when the repulsive magnets were configured so as to pass closely by each other, the design was insensitive to changes in input frequency for power harvesting.

Sneller *et al* [48] and Mann and Owens [49] used magnetic repulsion of a magnet oscillating along the axis of a tube to create bistability; induction of the oscillating magnet through a surrounding coil served as the electromechanical conversion mechanism. Inducement of high-energy orbits was shown to provide substantial improvement in output power. A similar device exists in the literature without the destabilizing mechanism [5], but no direct performance comparison was made to show the advantage of the bistable harvester to the monostable Duffing oscillator counterpart.

Karami *et al* [52] employed a circular array of cantilevered piezoelectric beams with magnetic tip masses that were activated via a vertical-axis windmill having a shaft at the center of the beam array. Repulsive magnets were connected to the windmill shaft. Thus, as the windmill rotated, the base-fixed piezoelectric beams were excited by the repulsion of the tip magnets and the revolving magnets. This concept is unique compared to the other studies in this review since mechanical vibration is not the input excitation mechanism. An optimum angular velocity of the windmill was observed which most excited the array of piezoelectric beams; the optimum rate was found also to be a function of the gap between the repulsing magnets. The dynamics of the system were found to be highly complex in regard to the magnetic repelling force per rotation of the windmill. Advantages of the proposed device as compared to other piezoelectric windmills found in the literature [55, 56] are the lowered required wind speed to start up and the broad range of wind speeds useful for power harvesting.

4.3. Mechanical bistability

Mechanical design and loading offer a variety of means by which to induce bistability into energy harvesters, including methods inspired by biological structures [84]. A readily adjustable bistability mechanism is a clamped–clamped beam buckled by an axial load. The post-buckled beam therefore snaps from one stable state to the other when excited by enough input excitation. To make this concept useful for energy harvesting, piezoelectric patches are applied to the beam such that oscillations of the beam will strain the piezoelectric layers, as depicted in figure 3(c). This configuration was earlier proposed by Baker *et al* [57], where experiments of frequency-swept excitation were conducted to validate the hypothesis that the bistability could achieve greater levels of broadband power than the same beam without a destabilizing axial load.

Cottone *et al* [30] compared this bistable harvester design with the unbuckled configuration when excited by exponentially correlated noise. The output RMS voltage was increased by an order of magnitude for the bistable device as compared with the unbuckled sample. Experiments and numerical modeling showed an optimum input acceleration level exists for the bistable harvester; this finding contrasts with linear harvesters, for which increases in input acceleration proportionally increase the harvested power.

Masana and Daqaq [58–60] have carried out detailed studies of the post-buckled piezoelectric beam. The depth of the double-well potential was found to play a crucial role in the benefit of the bistable harvester. The weaker the bistability (that is, maintaining an axial load close to the critical buckling load), the less advantage would be attained as compared with the unbuckled beam since the restoring force potentials were not substantially different. However, the advantage of the bistable device over the linear device was not uniform, with the exception at very low frequencies when the bistable harvester was excited into high-energy orbits but the linear harvester was weakly excited. Superharmonic dynamics were specifically considered in a series of comparable tests and simulations [60]. This uniquely nonlinear dynamic regime was found to provide a substantial increase in output power as compared to the linear harvester, so long as the device maintained the high-energy orbit and did not degenerate into a coexisting low-energy stable state.

The bistability of a plate may be generated by composite laminate lay-up. The variation in ply orientation and geometry allow for a unique tailoring of the two stable equilibria natural frequencies. Additionally, the spread and distribution of the piezoelectric patches on the plate surfaces may serve as optimization parameters for energy harvesting. Following initial modeling analyses and experimental studies to illustrate the potential of the bistable harvester plate [33, 34], Betts *et al* [61] determined optimal lay-up configurations and aspect ratios for energy harvesting. It was found that square plates were the optimal lamina shape despite the greater out-of-plane deflections attainable by higher aspect ratios. This was attributed to the unbiased nature of the square shape in vibrating between the two stable states, whereas plates with aspect ratios $\neq 1$ exhibit a preference to one of the stable modes that inhibits snap-through.

Bistability induced by an applied axial load can alternatively be achieved using an inverted clamped piezoelectric beam and a tip mass selected so as to buckle the system. This configuration was extensively explored by Friswell *et al* [62], who demonstrated the advantages of the design for extremely low-frequency vibration environments. The inverted beam configuration was not easily excited to interwell oscillation in experiment. As such, designing the tip mass so that the beam was subjected to a near-critical buckling load produced the most favorable results.

Jung and Yun [63, 64] studied frequency up-conversion methods that exploit the impulsive snap-through behavior of a buckled beam. An array of linear cantilevered piezoelectric harvesters was attached to a post-buckled clamped–clamped beam. Tests showed that very low frequency excitation (one order of magnitude less in frequency than the natural frequency of the attached linear cantilevers) was sufficient to yield consistent power output before the ring down decayed substantially. An optimum excitation frequency was measured and found to be approximately one-third of the natural frequency of the attached cantilevers. Power harvesting around this excitation frequency was found to be much less sensitive to frequency changes as compared with the single cantilevers excited at their natural frequency; this indicates another advantage of bistable energy harvesting in providing a broad harvesting bandwidth.

5. Challenges in bistable energy harvesting

Despite the documented potential and advantages of bistable harvesters, it has been shown that there is room for improvement and advancement for these nonlinear devices. This section summarizes several of the key remaining challenges and some proposed solutions.

5.1. Maintaining high-energy orbits

One principle challenge is the appropriate means by which to maintain high-energy orbits for maximum power harvesting performance. Erturk and Inman [26] demonstrated that a mechanical shock to the system could help the bistable harvester recover a high-energy orbit when it was earlier in intrawell or chaotic vibration. Masuda and Senda [65] observed that a sudden change in external circuit impedance could destabilize the intrawell vibration, returning the oscillator into a high-energy orbit. Sebald *et al* [66] described a similar technique whereby an impulsive voltage could be applied in the harvesting circuit to achieve the same objective. These methods are external interventions which require some form of monitoring and activation. As a result, the benefit of the approaches must be evaluated by how much energy is expended relative to the overall harvested power.

Understanding of the excitation characteristics required to induce interwell dynamics is an area of rigorous mathematical investigation. Melnikov theory [41], period-doubling bifurcation [67], and evaluation of Lyapunov exponents [40, 68] are all candidate efforts to quantify the threshold between intrawell and interwell oscillations. Since sustaining high-energy orbits is critical to maximizing energy harvesting performance, and interventionary measures as mentioned above reduce the net output, a clear knowledge of what design and operational parameters are necessary to maintain high-energy orbits is required. Further analytical investigation and subsequent experimental validation are still necessary to better characterize the sustainability of high-energy bistable dynamics.

5.2. Operation in a stochastic vibratory environment

Realistic vibration environments for which energy harvesters are employed are likely composed of multi-frequency harmonics as well as a substantial proportion of low-pass filtered noise. Although it has been shown that a bistable device may outperform the linear equivalent in stochastic environments, this conclusion draws on the assumption that the bistable harvester exhibits interwell vibrations. Should intrawell vibrations be observed, it has been proposed to utilize the random excitation component in tandem with small coherent sinusoidal excitation to induce stochastic resonance [31, 32]. McInnes *et al* [17] demonstrated that this combination could be successfully exploited to induce interwell oscillations in a bistable harvester. After subtracting the theoretical active input power to modify the oscillator potential, the net power harvested was substantially greater than that harvested from passive intrawell vibrations. Litak *et al* [29] showed that a specific noise intensity maximizes the harvested power from bistable devices having a static potential-energy profile. Thus, in a realistic environment where the designer knows a typical stochastic vibration strength will dominate, the bistable harvester could be optimally designed. This conclusion was also verified analytically [10].

Chaotic oscillations of the bistable harvester are preferable to intrawell vibrations, but attaining high-energy orbits is the optimal goal. However, stochastic input vibrations in many environments may not contain the correct harmonics so as to sustain primarily periodic high-energy orbits; thus, aperiodic response may dominate a bistable energy harvester's behavior. The difficulty in harvesting a chaotic or aperiodic output voltage as useful electrical power has been recognized [26, 37], although contemporary work has provided some solutions with optimal stochastic energy harvesting controls [69]. The reality of ambient environmental vibration as compared to an ideal, stationary, and sinusoidal input makes for the ultimate challenge in practical energy harvesting. Fortunately, one of the advantages of bistable energy harvesters is their robustness to real-world unknowns [38]. At present, many of the initial investigations in stochastic energy harvesting encourage continued study and, in particular, experimental validation.

5.3. Coupled bistable harvesters

There has long been interest in the study of coupled systems exhibiting chaotic behavior for the means of advantageous synchronization and array control [70]. A recent study evaluated the dynamics of coupled underdamped bistable oscillators [71]. It was observed that stochastic resonance could be induced only with moderate damping regardless of coupling strength. However, optimal coupling and noise strength parameters could be determined which would yield greater signal-to-noise ratio than the uncoupled oscillators. While no decisive conclusions were offered to characterize the complex coupling dynamics, these findings show potential for the achievement of stochastic resonance in coupled harvesting systems.

To the authors' knowledge, only an initial study by Litak *et al* [72] has thus far considered the possibility of coupling bistable harvesters. In this report, a numerical investigation was carried out for swept single-frequency excitation with the harvesters coupled through a collective circuit. It was shown that identically excited bistable devices having different linear resonances could become unsynchronized, leading one harvester to vibrate chaotically where it would otherwise vibrate in high-energy orbits when uncoupled. While a single bistable oscillator exhibits an organized Poincare map when undergoing chaotic vibrations, see figure 2(e), the authors observed by case study that coupling may break down such a Poincare map structure. The mathematical challenges posed in the study of coupled bistable systems show this is still an area in need of investigation.

5.4. Performance metrics

There is no standing consensus in the literature on the preferred means by which to evaluate energy harvesting performance [11, 21, 22] since the type of input excitation considered and the spectral bandwidth of relevance vary. The challenges to developing consensus are exacerbated further by the complexity of the coupled external circuit to be studied, the associated losses therein, and whether or not additional performance metrics are weighted against the harvested power (e.g. vibration control and energy harvesting [73–75]). Nonlinear harvesters may also have multiple stable solutions for a given operating condition, making a steady-state performance metric ambiguous at best. The practice of many works in this review has been to directly compare tested or simulated results of bistable power harvesters with the linear equivalents. This approach is suitable for individual case studies but does not provide for general conclusions to be drawn. Thus, broadly applicable energy harvesting efficiency metrics or protocols remain to be determined.

6. Relation to bistable damping research

As indicated, earlier studies in energy harvesting simplified analyses by utilizing only the mechanical governing equation [16–19]. This presumes that the net mechanical energy dissipated would serve as a theoretical ceiling on the harvested power and that electromechanical coupling is equivalent to additional velocity-proportional damping. If this perspective is maintained, contemporaneous work in bistable vibration damping should be recognized and noted for analytical results that do not have counterparts in existing bistable energy harvesting research but which may be useful for future studies.

Avramov and Mikhlin [76] considered the vibration absorption capability of a bistable snap-through truss attached to a main elastic system. The method of nonlinear normal modes (NNM) was employed and found to be accurate as compared with direct numerical integration when the snap-through oscillator exhibited intrawell oscillations. Once snap-through occurred, trajectories predicted by NNM diverged from simulation but relative modal amplitudes remained consistent. It was found that localization of the NNM could be attained within the snap-through truss, thus maximizing vibration energy transfer to that element. The localized NNM was shown to be stable using MMS. Gendelman and Lamarque [77] also used MMS to determine dynamic manifolds for a bistable oscillator and coupled host vibrating oscillator. Three distinct zones were determined that indicated efficient energy pumping into the bistable device, energy dissipated via intrawell vibration, and transient chaos. Numerically integrated simulations verified the regimes and the approximate bounds amongst the predicted manifolds. These results may provide insight and initial direction to coupled bistable energy harvesting research.

Bistable vibration damping studies have a variety of protocols regarding efficiency and energy transfer [78] which may be of benefit in the ultimate determination of energy harvesting metrics. Johnson *et al* [79] applied a loss factor criteria to a bistable snap-through device; while convergence of the measure was shown, it was suggested that it may not always be used when the oscillator undergoes chaos vibration. It was also illustrated that bistable devices can be used for designing adaptive damping with respect to input amplitude and frequency [79]. Studies in micro- and nano-metamaterials having bistable inclusions for increased vibration and acoustic damping use a variety of methods for material performance evaluation [80–83]. These concepts should be considered in the resolution of energy harvesting metrics generally, and may provide clear, comparable evidence of the dramatic advantages of bistable harvesters thus displayed in analyses and experiments.

7. Concluding remarks

The benefits of exploiting bistable nonlinearities in vibration energy harvesting have been the impetus for much recent research. A breadth of studies have been undertaken to shed light on the intricate electromechanical dynamics and to provide experimental evidence of the predicted benefits. The various bistable harvester designs thus far studied have relied heavily on magnetic attraction, magnetic repulsion, and mechanical loading to induce the bistability. Other investigations have employed the bistability mechanism itself as a novel excitation source for frequency up-conversion applications. Depending on the excitation environment, either the periodic excitation of bistable interwell dynamics or a number of frequency up-conversion techniques can be utilized to provide practical energy harvesting output, exemplifying the versatility of bistable harvester designs.

On the whole, bistable harvesters are an improvement upon their linear counterparts in steady-state vibration environments and have been analytically and experimentally shown to provide as much as an order of magnitude increase in harvested energy. The benefits or disadvantages of bistable devices due to stochastic excitation have not yet been conclusively determined and a genuine need remains to better understand the potential of vibration energy harvesting in random excitation environments. A number of advanced topics have only begun to be explored, such as the accurate and reliable prediction of high-energy bistable dynamics for maximum harvesting performance and the opportunities provided for by multi-degree-of-freedom systems containing bistable elements. Concentrated efforts are required to answer the questions involved for realistic vibration energy harvesting with bistable devices. Researchers in the field may also find inspiration from contemporaneous work in advanced metamaterials and bistable vibration damping. To date, vibration energy harvesting studies have drawn upon the expertise from members among a number of research communities in order to solve the problems of optimum device development and analytical assessment. Continued collaborative efforts will be necessary to formulate novel solutions and implementations to the successful utilization of bistable systems as an effective and robust energy harvesting platform.

References

[1] Roundy S, Wright P K and Rabaey J 2003 A study of low level vibrations as a power source for wireless sensor nodes *Comput. Commun.* **26** 1131–44

[2] duToit N E, Wardle B L and Kim S G 2005 Design considerations for MEMS-scale piezoelectric mechanical vibration energy harvesters *Integr. Ferroelectr.* **71** 121–60

[3] Tang L, Yang Y and Soh C K 2010 Toward broadband vibration-based energy harvesting *J. Intell. Mater. Syst. Struct.* **21** 1867–97

[4] Zhu D, Tudor M J and Beeby S P 2010 Strategies for increasing the operating frequency range of vibration energy harvesters: a review *Meas. Sci. Technol.* **21** 022001

[5] Mann B P and Sims N D 2009 Energy harvesting from the nonlinear oscillations of magnetic levitation *J. Sound Vib.* **319** 515–30

[6] Daqaq M F 2010 Response of uni-modal Duffing-type harvesters to random forced excitations *J. Sound Vib.* **329** 3621–31

[7] Sebald G, Kuwano H, Guyomar D and Ducharne B 2011 Experimental Duffing oscillator for broadband piezoelectric energy harvesting *Smart Mater. Struct.* **20** 102001

[8] Umeda M, Nakamura K and Ueha S 1996 Analysis of transformation of mechanical impact energy to electrical energy using a piezoelectric vibrator *Japan. J. Appl. Phys.* **35** 3267–73

[9] Gu L and Livermore C 2011 Impact-driven, frequency up-converting coupled vibration energy harvesting device for low frequency operation *Smart Mater. Struct.* **20** 045004

[10] Daqaq M F 2011 Transduction of a bistable inductive generator driven by white and exponentially correlated Gaussian noise *J. Sound Vib.* **330** 2554–64

[11] Pellegrini S P, Tolou N, Schenk M and Herder J L 2012 Bistable vibration energy harvesters: a review *J. Intell. Mater. Syst. Struct.* doi:10.1177/1045389X12444940

[12] Tseng W Y and Dugundji J 1971 Nonlinear vibrations of a buckled beam under harmonic excitations *J. Appl. Mech.* **38** 467–76

[13] Holmes P 1979 A nonlinear oscillator with a strange attractor *Phil. Trans. R. Soc.* A **292** 419–48

[14] Moon F C and Holmes P J 1979 A magnetoelastic strange attractor *J. Sound Vib.* **65** 275–96

[15] Kovacic I and Brennan M J (ed) 2011 *The Duffing Equation: Nonlinear Oscillators and their Behaviour* 1st edn (Chichester: Wiley)

[16] Shahruz S M 2008 Increasing the efficiency of energy scavengers by magnets *J. Comput. Nonlinear Dyn.* **3** 041001

[17] McInnes C R, Gorman D G and Cartmell M P 2008 Enhanced vibrational energy harvesting using nonlinear stochastic resonance *J. Sound Vib.* **318** 655–62

[18] Ramlan R, Brennan M J, Mace B R and Kovacic I 2010 Potential benefits of a non-linear stiffness in an energy harvesting device *Nonlinear Dyn.* **59** 545–58

[19] Beeby S P, Tudor M J and White N M 2006 Energy harvesting vibration sources for microsystems applications *Meas. Sci. Technol.* **17** R175–95

[20] Poulin G, Sarraute E and Costa F 2004 Generation of electrical energy for portable devices, comparative study of an electromagnetic and a piezoelectric system *Sensors Actuators* A **116** 461–71

[21] Shu Y C and Lien I C 2006 Efficiency of energy conversion for a piezoelectric power harvesting system *J. Micromech. Microeng.* **16** 2429–38

[22] Liao Y and Sodano H A 2009 Structural effects and energy conversion efficiency of power harvesting *J. Intell. Mater. Syst. Struct.* **20** 505–14

[23] Mann B P and Sims N D 2010 On the performance and resonant frequency of electromagnetic induction energy harvesters *J. Sound Vib.* **329** 1348–61

[24] Stanton S C, McGehee C C and Mann B P 2010 Nonlinear dynamics for broadband energy harvesting: investigation of a bistable piezoelectric inertial generator *Physica* D **239** 640–53

[25] Karami M A and Inman D J 2011 Equivalent damping and frequency change for linear and nonlinear hybrid vibrational energy harvesting systems *J. Sound Vib.* **330** 5583–97

[26] Erturk A and Inman D J 2011 Broadband piezoelectric power generation on high-energy orbits of the bistable Duffing oscillator with electromechanical coupling *J. Sound Vib.* **330** 2339–53

[27] Daqaq M F 2012 On intentional introduction of stiffness nonlinearities for energy harvesting under white Gaussian excitations *Nonlinear Dyn.* **69** 1063–79

[28] Cottone F, Vocca H and Gammaitoni L 2009 Nonlinear energy harvesting *Phys. Rev. Lett.* **102** 080601

[29] Litak G, Friswell M I and Adhikari S 2010 Magnetopiezoelastic energy harvesting driven by random excitations *Appl. Phys. Lett.* **96** 214103

[30] Cottone F, Gammaitoni L, Vocca H, Ferrari M and Ferrari V 2012 Piezoelectric buckled beams for random vibration energy harvesting *Smart Mater. Struct.* **21** 035021

[31] Gammaitoni L, Hänggi P, Jung P and Marchesoni F 1998 Stochastic resonance *Rev. Mod. Phys.* **70** 223–87

[32] Wellens T, Shatokhin V and Buchleitner A 2004 Stochastic resonance *Rep. Prog. Phys.* **67** 45–105

[33] Arrieta A F, Neild S A and Wagg D J 2009 Nonlinear dynamic response and modeling of a bi-stable composite plate for applications to adaptive structures *Nonlinear Dyn.* **58** 259–72

[34] Arrieta A F, Hagedorn P, Erturk A and Inman D J 2010 A piezoelectric bistable plate for nonlinear broadband energy harvesting *Appl. Phys. Lett.* **97** 104102

[35] Arrieta A F, Neild S A and Wagg D J 2011 On the cross-well dynamics of a bi-stable composite plate *J. Sound Vib.* **330** 3424–41

[36] Shaw A D and Carrella A 2012 *Force Displacement Curves of a Snapping Bistable Plate* (*Topics in Nonlinear Dynamics* vol 3) ed D Adams, G Kerschen and A Carrella (New York: Springer) pp 191–7

[37] Stanton S C, Owens B A M and Mann B P 2012 Harmonic balance analysis of the bistable piezoelectric inertial generator *J. Sound Vib.* **331** 3617–27

[38] Mann B P, Barton D A W and Owens B A M 2012 Uncertainty in performance for linear and nonlinear energy harvesting strategies *J. Intell. Mater. Syst. Struct.* **23** 1451–60

[39] Guckenheimer J and Holmes P J 1983 *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields* (New York: Springer)

[40] Wiggins S 2003 *Introduction to Applied Nonlinear Dynamical Systems and Chaos* (New York: Springer)

[41] Stanton S C, Mann B P and Owens B A M 2012 Melnikov theoretic methods for characterizing the dynamics of a bistable piezoelectric inertial generator in complex spectral environments *Physica* D **241** 711–20

[42] Andò B, Baglio S, Trigona C, Dumas N, Latorr L and Nouet P 2010 Nonlinear mechanism in MEMS devices for energy harvesting applications *J. Micromech. Microeng.* **20** 125020

[43] Ferrari M, Ferrari V, Guizzetti M, Andò B, Baglio S and Trigona C 2010 Improved energy harvesting from wideband vibrations by nonlinear piezoelectric converters *Sensors Actuators* A **162** 425–31
[44] Picchini U SDE toolbox: simulation and estimation of stochastic differential equations with MATLAB (Online), available from: http://sdetoolbox.sourceforge.net
[45] Cohen N, Bucher I and Feldman M 2012 Slow–fast response decomposition of a bi-stable energy harvester *Mech. Syst. Sig. Process.* **31** 29–39
[46] Erturk A, Hoffmann J and Inman D J 2009 A piezomagnetoelastic structure for broadband vibration energy harvesting *Appl. Phys. Lett.* **94** 254102
[47] Galchev T, Kim H and Najafi K 2011 Micro power generator for harvesting low-frequency and nonperiodic vibrations *J. Microelectromech. Syst.* **20** 852–66
[48] Sneller A J, Owens B A and Mann B P 2010 Experimental investigations of a bistable energy harvester *Proc. IMAC-XXVII (Jacksonville, FL)* pp 211–7
[49] Mann B P and Owens B A 2010 Investigations of a nonlinear energy harvester with a bistable potential well *J. Sound Vib.* **329** 1215–26
[50] Lin J T and Alphenaar B 2010 Enhancement of energy harvested from a random vibration source by magnetic coupling of a piezoelectric cantilever *J. Intell. Mater. Syst. Struct.* **21** 1337–41
[51] Vocca H, Neri I, Travasso F and Gammaitoni L 2012 Kinetic energy harvesting with bistable oscillators *Appl. Energy* **97** 771–6
[52] Karami M A, Farmer J R and Inman D J 2013 Parametrically excited nonlinear piezoelectric compact wind turbine *Renew. Energy* **50** 977–87
[53] Tang L, Yang Y and Soh C K 2012 Improving functionality of vibration energy harvesters using magnets *J. Intell. Mater. Syst. Struct.* **23** 1433–49
[54] Ramlan R, Brennan M J, Mace B R and Burrow S G 2012 On the performance of a dual-mode non-linear vibration energy harvesting device *J. Intell. Mater. Syst. Struct.* **23** 1423–32
[55] Myers R, Vickers M, Kim H and Priya S 2007 Small scale windmill *Appl. Phys. Lett.* **90** 054106
[56] Tien C M T and Goo N S 2010 Use of a piezo-composite generating element for harvesting wind energy in an urban region *Aircr. Eng. Aerosp. Technol.* **82** 376–81
[57] Baker J, Roundy S and Wright P 2005 Alternative geometries for increasing power density in vibration energy scavenging for wireless sensor networks *Proc. 3rd Int. Energy Conversion Engineering Conf. (San Francisco)* AIAA 2005-5617
[58] Masana R and Daqaq M F 2011 Electromechanical modeling and nonlinear analysis of axially loaded energy harvesters *J. Vib. Acoust.* **133** 011007
[59] Masana R and Daqaq M F 2011 Relative performance of a vibratory energy harvester in mono- and bi-stable potentials *J. Sound Vib.* **330** 6036–52
[60] Masana R and Daqaq M F 2012 Energy harvesting in the super-harmonic frequency region of a twin-well oscillator *J. Appl. Phys.* **111** 044501
[61] Betts D N, Kim H A, Bowen C R and Inman D J 2012 Optimal configurations of bistable piezo-composites for energy harvesting *Appl. Phys. Lett.* **100** 114104
[62] Friswell M I, Ali S F, Bilgen O, Adhikari S, Lees A W and Litak G 2012 Non-linear piezoelectric vibration energy harvesting from a vertical cantilever beam with tip mass *J. Intell. Mater. Syst. Struct.* **23** 1505–21
[63] Jung S M and Yun K S 2010 A wideband energy harvesting device using snap-through buckling for mechanical frequency-up conversion *2010 IEEE 23rd Int. Conf. Proc. Micro Electro Mechanical Systems (MEMS) (Hong Kong)* pp 1207–10
[64] Jung S M and Yun K S 2010 Energy-harvesting device with mechanical frequency-up conversion for increased power efficiency and wideband operation *Appl. Phys. Lett.* **96** 111906
[65] Masuda A and Senda A 2012 Stabilization of a wide-band nonlinear vibration energy harvester by using a nonlinear self-excitation circuit *Proc. SPIE* **8341** 83411B
[66] Sebald G, Kuwano H, Guyomar D and Ducharne B 2011 Simulation of a Duffing oscillator for broadband piezoelectric energy harvesting *Smart Mater. Struct.* **20** 075022
[67] Asfar K R and Masoud K K 1992 On the period-doubling bifurcations in the Duffing's oscillator with negative linear stiffness *J. Vib. Acoust.* **114** 489–94
[68] Nayfeh A H and Balachandran B 1995 *Applied Nonlinear Dynamics: Analytical, Computational, and Experimental Methods* (Weinheim: Wiley)
[69] Scruggs J T 2009 An optimal stochastic control theory for distributed energy harvesting networks *J. Sound Vib.* **320** 707–25
[70] Kurths J, Boccaletti S, Grebogi C and Lai Y C 2003 Introduction: control and synchronization in chaotic dynamical systems *Chaos* **13** 126–7
[71] Kenfack A and Singh K P 2010 Stochastic resonance in coupled underdamped bistable systems *Phys. Rev.* E **82** 046224
[72] Litak G, Friswell M I, Kitio Kwuimy C A, Adhikari S and Borowiec M 2012 Energy harvesting by two magnetopiezoelastic oscillators with mistuning *Theor. Appl. Mech. Lett.* **2** 043009
[73] Tang X and Zuo L 2011 Simulation and experiment validation of simultaneous vibration control and energy harvesting from buildings using tuned mass dampers *2011 American Control Conf. (San Francisco)* pp 3134–9
[74] Cassidy I L, Scruggs J T, Behrens S and Gavin H P 2011 Design and experimental characterization of an electromagnetic transducer for large-scale vibratory energy harvesting applications *J. Intell. Mater. Syst. Struct.* **22** 2009–24
[75] Harne R L 2012 Theoretical investigations of energy harvesting efficiency from structural vibrations using piezoelectric and electromagnetic oscillators *J. Acoust. Soc. Am.* **132** 162–72
[76] Avramov K V and Mikhlin Y V 2004 Snap-through truss as a vibration absorber *J. Vib. Control* **10** 291–308
[77] Gendelman O V and Lamarque C H 2005 Dynamics of linear oscillator coupled to strongly nonlinear attachment with multiple states of equilibrium *Chaos Solitons Fractals* **24** 501–9
[78] Vakakis A F, Gendelman O V, Bergman L A, McFarland D M, Kerschen G and Lee Y S 2008 *Nonlinear Targeted Energy Transfer in Mechanical and Structural Systems* (Dordrecht: Springer)
[79] Johnson D R, Thota M, Semperlotti F and Wang K W 2012 A bi-stable oscillator for increasing damping and providing passive adaptability *Proc. SPIE* **8341** 83410A
[80] Lakes R S, Lee T, Bersie A and Wang Y C 2001 Extreme damping in composite materials with negative-stiffness inclusions *Nature* **410** 565–7
[81] Haberman M R 2007 Design of high loss viscoelastic composites through micromechanical modeling and decision based materials design *PhD Dissertation* Georgia Institute of Technology
[82] Kashdan L, Seepersad C C, Haberman M and Wilson P S 2012 Design, fabrication, and evaluation of negative stiffness elements using SLS *Rapid Prototyping J.* **18** 194–200
[83] Dong L and Lakes R S 2012 Advanced damper with negative structural stiffness elements *Smart Mater. Struct.* **21** 075026
[84] Kim G W and Kim J 2013 Compliant bistable mechanism for low frequency vibration energy harvester inspired by auditory hair bundle structures *Smart Mater. Struct.* **22** 014005