



## Methods in Vehicle Mass and Road Grade Estimation

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### ABSTRACT

Dynamic vehicle loads play critical roles for automotive controls including battery management, transmission shift scheduling, distance-to-empty predictions, and various active safety systems. Accurate real-time estimation of vehicle loads such as those due to vehicle mass and road grade can thus improve safety, efficiency, and performance. While several estimation methods have been proposed in literature, none have seen widespread adoption in current vehicle technologies despite their potential to significantly improve automotive controls. To understand and bridge the gap between research development and wider adoption of real-time load estimation, this paper assesses the accuracy and performance of four estimation methods that predict *vehicle mass* and/or *road grade*. These include recursive least squares (RLS) with multiple forgetting factors; extended Kalman filtering (EKF); a dynamic grade observer (DGO); and a method developed by this research: parallel mass and grade (PMG) estimation using a longitudinal accelerometer.

The estimation methods and models were constructed, numerous vehicle tests were performed, and data was evaluated off-line by the estimation approaches. It is found that RLS and EKF yield estimates within 5% of their actual values if provided initial values close to true initial states. To improve estimation when inaccurate initial values are provided, a mass selection algorithm is proposed that determines mass based on converged values from concurrently-operating EKF estimators. Its potential for accurate mass and grade estimation is demonstrated. PMG estimation provides the most reliable and accurate results, and demonstrates the greatest potential for successful real-time implementation to advance the performance, economy, and reliability of future vehicle controls.

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### INTRODUCTION

Changes in inertial parameters of road vehicles have a significant effect on ride quality, performance, handling, and energy efficiency. As automotive subsystems become increasingly automated, accurate knowledge of inertial parameters is critical to ensuring passenger safety while delivering improved vehicle performance. Therefore, it is evident that vehicle mass and road grade are two parameters related to critical loads that must be estimated accurately and reliably.

For successful operation, various controllers require accurate identification of vehicle mass, but passenger and cargo loading may cause variations in mass of up to 50% for small cars from trip to trip. The difference between accurate real-time identification of mass and its assumption of constant value

throughout all operating conditions directly reflects a reduction in the accuracy and performance of automotive controls. Meanwhile, light trucks and cargo vans may vary in mass by 400% depending on payload conditions. Vehicle mass is directly related to tire normal forces, which influence lateral and longitudinal tire force generation [1]. Consequently, active safety technologies such as anti-lock braking systems, collision avoidance, and stability control can benefit from accurate knowledge of vehicle mass [2, 3]. It has been shown that for typical passenger vehicles, a 10% increase in vehicle mass corresponds to a 2.4% to 4.1% increase in energy usage [4]. Neglecting mass variations negatively affects the accuracy of distance-to-empty predictions and compromises effective battery management of hybrid electric or battery electric vehicles. The presence of road grade can introduce a

significant load to the powertrain. Transmission shift scheduling, adaptive cruise control, and hill start assist [5] are technologies that benefit from accurate knowledge of road grade.

While various controllers on-board a vehicle can independently estimate relevant parameters, it may be advantageous to have a single, accurate, on-line estimate of vehicle mass and road grade applied across controller platforms. This avoids redundant computations and conflicting parameter estimates, which is crucial when a supervisory controller is coordinating the behavior of multiple control systems [6]. While there have been numerous studies for inertial load estimation which employ various prediction methods and modeling approaches [3, 7, 8, 9, 10], many previous efforts toward on-line mass and grade estimation using existing on-board sensor systems have been based on vehicle longitudinal dynamics models due to the many common driving scenarios for which such models apply. Estimation approaches include recursive least squares (RLS) with multiple forgetting factors [11, 12, 13], extended Kalman filtering (EKF) [5], a dynamic grade observer (DGO) [14] requiring only longitudinal acceleration and an estimate of powertrain torque, and grade estimation using kinematic information provided by a longitudinal accelerometer [15]. As developed in this paper, kinematic information may also be used to estimate mass without explicit calculation of road grade: a *parallel* mass and grade estimation.

Mass and grade estimation methods reported in literature are not widely employed in production automotive controls at present. In addition, there are few reported efforts which directly compare such estimation approaches using data that represents a range of driving conditions [13], nor have they been comprehensively studied and compared using consistent experiments. A study to evaluate existing estimation schemes side-by-side may yield critical information regarding the accuracy and reliability of each method, may highlight impediments to their implementation which explain the lack of industry adoption, and may also uncover novel approaches by which the existing methods could be integrated or improved upon. Therefore, the goal of this research is to compare the existing real-time mass and grade estimation approaches, seek insights regarding suitable method implementation, and propose new solutions which advance the state-of-the-art of real-time vehicle load parameter estimation. The existing longitudinal dynamics-based RLS, EKF, and dynamic grade observer (DGO) methods are compared alongside a parallel mass and grade (PMG) estimation approach. Various driving data sets are employed to evaluate estimator reliability and accuracy. While global positioning (GPS)-based estimation approaches are reported in the literature [16, 17], comparison of such approaches is omitted because GPS information is not necessarily available or reliable as compared to vehicle sensors requiring only local access of dynamic information.

The remainder of this paper is structured as follows: The first section introduces the longitudinal dynamics model used by all the estimation methods. Then, the on-line estimation methods compared in this study are described. Experiments to evaluate

these methods are detailed and the results are presented. A mass selection algorithm for use with simultaneous mass and grade estimation methods is proposed and validated using the convergence of parameters under several different initial conditions. Then, the independent mass and grade estimation methods are assessed. Finally, the last section summarizes the conclusions and discusses future research opportunities.

## LONGITUDINAL DYNAMICS MODEL

For typical driving scenarios, longitudinal motion is dominant and its measurement provides a near-continuous data set that can be used for real-time estimates of vehicle loads. Interruptions of estimation may occur during gear shifts, braking, and periods of significant vehicle yaw and roll because such events are not easily captured by conventional longitudinal dynamics modeling. The forces influencing automotive longitudinal dynamics are shown in Figure 1. The tractive force due to tire-road interactions is  $F_w$ . This study assumes no tire slip and that the tractive force may be accurately determined by  $F_w = T_w/r_w$ , where  $T_w$  is wheel torque and  $r_w$  is tire radius. Forces opposing longitudinal motion include aerodynamic drag  $F_{aero}$ , rolling resistance  $F_\mu$ , grade forces  $F_{grade}$  and braking  $F_{fb}$  if present.

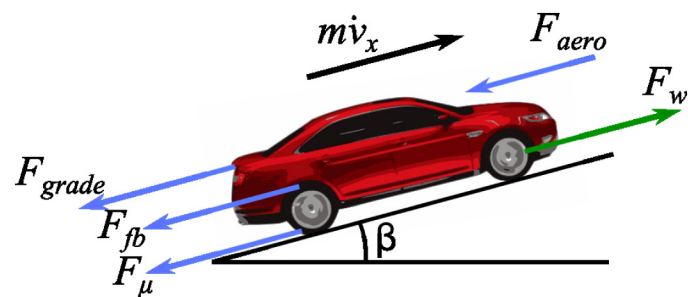


Figure 1. Illustration of forces influencing vehicle longitudinal dynamics, including tractive force  $F_w$ , aerodynamics drag force  $F_{aero}$ , grade force  $F_{grade}$ , rolling resistance force  $F_\mu$  and friction brake force  $F_{fb}$ .

From these assumptions, the longitudinal dynamics of a vehicle may be expressed through Newton's 2nd law as

$$m\dot{v}_x = \frac{T_w}{r_w} - \frac{1}{2}\rho C_d A_f v_x^2 - mg(\mu \cos \beta + \sin \beta) \quad (1)$$

where  $m$  is vehicle mass;  $v_x$  is longitudinal velocity;  $\rho$  is the density of air;  $C_d$  is the vehicle's drag coefficient;  $A_f$  is the vehicle's frontal area,  $g$  is the gravitational constant,  $\mu$  is the vehicle's rolling resistance, and  $\beta$  is road grade. The wheel torque  $T_w$  may be computed from a measurement on the driveline, combined with a driveline model that considers losses due to friction and inertial effects. Making the substitution  $\beta_\mu = \tan^{-1} \mu$  allows equation (1) to be written as:

$$\dot{v}_x = \left( \frac{T_w}{r_w} - \frac{1}{2}\rho C_d A_f v_x^2 \right) \frac{1}{m} - \frac{g}{\cos(\beta_\mu)} \sin(\beta + \beta_\mu) \quad (2)$$

Equation (2) separates the loading effects of vehicle mass and road grade, and is used as the basic dynamic equation to which mass and grade estimation methods are applied in this study.

If the vehicle is equipped with a longitudinal accelerometer, road grade is computed from equation (3) relating measured acceleration  $a_x$  to the differentiated longitudinal velocity  $\dot{v}_x$ . On a horizontal surface, these two quantities are identical. However, if the accelerometer is tilted due to road grade or vehicle pitch, it also captures the vector projection of gravity along the axis of sensor measurement. If the accelerometer is tilted from the horizontal plane by  $\beta$  and vehicle pitching is absent, then  $g \sin(\beta)$  is the component of gravity projected onto its measurement axis [15, 17].

$$a_x = \dot{v}_x + g \sin(\beta) \quad (3)$$

## ON-LINE ESTIMATION METHODS

This section presents mass and grade estimation methods that are evaluated and compared in this study. They include simultaneous mass and grade estimation approaches (RLS and EKF), a dynamic grade observer (DGO) that only estimates grade, and a parallel mass and grade (PMG) estimator which uses a longitudinal accelerometer. For more detailed information on the development of these estimation methods, readers are encouraged to reference the applicable sources cited in the following subsections.

### Recursive Least Squares (RLS) with Multiple Forgetting Factors

Recursive least squares is a well-established method for estimating parameters in real time. It may be used with forgetting factors to account for parameters that are slowly-varying [18]. Vahidi et al. [12] presented a form of RLS that uses multiple forgetting factors for mass and grade, to reflect the fact that mass is constant while grade may slowly vary. Thus, this method *simultaneously* estimates vehicle mass and road grade. The approach is summarized below.

Equation (2) is rewritten more compactly as

$$\dot{v}_x = [\phi_1 \ \phi_2] \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \quad (4)$$

where

$$\phi_1 = \left( \frac{r_w}{r_w} - \frac{1}{2} \rho C_d A_f v_x^2 \right) \quad (5a)$$

$$\phi_2 = \frac{g}{\cos(\beta_\mu)} \quad (5b)$$

$$\theta_1 = \frac{1}{m} \quad (5c)$$

$$\theta_2 = \sin(\beta + \beta_\mu) \quad (5d)$$

Two different forgetting factors,  $\lambda_1, \lambda_2 < 1$ , are applied to the RLS update equations for mass and grade respectively. These forgetting factors influence parameter update gains as well as convergence of the covariance matrix. A forgetting factor of 1 is used when the estimated parameter is assumed to be constant, in which case all prior information is applicable for identifying the parameter's current value. Consequently, the mass forgetting factor  $\lambda_1$  should be defined very close to unity because mass is assumed to be essentially constant for the duration of a trip. The forgetting factor for grade  $\lambda_2$  should be less than  $\lambda_1$  since grade is more likely to vary over the course of a continuous driving event as compared to vehicle mass loading. Together  $\lambda_{1,2}$  represent tuning parameters for RLS mass and grade estimation and detailed evaluation of their influence may be found in [12].

### Extended Kalman Filter (EKF)

Kalman filters are used extensively in state estimation, and can be implemented for *simultaneous* mass and grade estimation if these two parameters are treated as system states with low variance. Due to the effects of aerodynamic drag, equation (1) is not linear in  $v_x$ . Thus the extended Kalman filter [19] must be used to linearize the system about an operating velocity. An EKF discretized by distance to estimate constant mass and grade was presented by Winstead et al. [5] in the context of adaptive cruise control, but was not explored for varying grade as is investigated in this study. In this paper a discrete-time filter is formulated using an Euler approximation for the following state vector at time  $k$ .

$$x(k) = \begin{bmatrix} v_x(k) \\ \theta_1(k) \\ \theta_2(k) \end{bmatrix} \quad (6)$$

$v_x$  is longitudinal vehicle velocity while  $\theta_1$  and  $\theta_2$  are defined in equation (5). The system propagates in one time step as

$$x(k+1) = x(k) + T_s f(x(k), u(k)) + w(k) \quad (7)$$

with differentiate state transition model

$$f(x(k), u(k)) = \begin{bmatrix} \phi_1(k)\theta_1(k) + \phi_2(k)\theta_2(k) \\ 0 \\ 0 \end{bmatrix} \quad (8)$$

and process noise

$$w(k) = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \quad (9)$$

$T_s$  is the sampling period, and process noise  $w(k)$  is assumed to be zero-mean with diagonal covariance matrix  $Q$ . Velocity is the observable quantity, thus the observation model is

$$y(k+1) = Hx(k+1) + v(k+1) \quad (10)$$

The observation matrix  $H$  is  $[1 \ 0 \ 0]$  and  $v(k+1)$  represents observation noise with zero mean and covariance  $R$ . The model described in [equations \(7,8,9,10,11\)](#) may be implemented with the extended Kalman filter. A thorough explanation of the filter, including linearization and equations for the prediction and update step, is found in [\[13\]](#).

### Dynamic Grade Observer (DGO)

If mass is known, grade is the only unknown parameter in [equation \(1\)](#). McIntyre et al. [\[14\]](#) formulate vehicle longitudinal dynamics as in the manner of [equation \(4\)](#) and propose an observer to update estimates of grade, related to  $\theta_2$ . Therefore, the DGO method *only estimates grade* and does not independently provide a vehicle mass estimate.

Consider the observation error between measured and modeled velocity

$$e = v - \hat{v} \quad (11)$$

Modeled velocity is found by integrating modeled acceleration based on estimated road grade, and the error derivative

$$\dot{e} = \dot{v} - \dot{\hat{v}} \quad (12)$$

The following observer is proposed

$$\dot{e} + e = \phi_2 \dot{\theta}_2 - (k_1 + 1)(\dot{e} + e) + k_2 \text{sgn}(e) + \dot{e} \quad (13)$$

$k_1$  and  $k_2$  are observer gains. The proof and conditions for convergence, and a demonstration of the DGO in combination with an RLS estimator for mass, can be found in [\[14\]](#).

### Parallel Mass and Grade (PMG) Estimator Using a Longitudinal Accelerometer

A parallel mass and grade estimation approach is devised here which *independently* estimates mass and grade when a longitudinal accelerometer is available on-board the vehicle. An increasing number of vehicles on the road today are equipped with reliable and accurate longitudinal accelerometers. These sensors capture a vehicle's longitudinal acceleration but also measure the component of gravity along the measurement axis

due to tilting caused by grade and body pitch, as described in [equation \(3\)](#). This allows direct extraction of road grade information with only two signals assuming that vehicle pitching is absent from the vehicle dynamics, as has been earlier recognized in the literature [\[15, 17\]](#).

Because the accelerometer signal is affected by noise factors including vehicle bounce and pitch motions, [equation \(3\)](#) may not always be accurate for computation of road grade. To mitigate these effects, this research utilizes a weighted moving average filter based on signal variance over a buffer length. Since grade is a slowly varying parameter, large signal variance during a short buffer period indicates significant bounce and pitch motion, and the grade estimate update during this time is treated with lower confidence. In this manner, grade is *independently estimated* using available vehicle data and the accelerometer. During periods of constant acceleration, vehicle pitching may cause the grade estimate to be biased. Although the proposed PMG approach neglects this behavior, a pitch correction factor may be determined either experimentally by measuring steady state pitch angles under a range of constant accelerations on flat ground. Alternatively, this pitch correction factor can be determined analytically as a function of longitudinal acceleration if suspension parameters are known.

Once an estimate of grade is obtained, a longitudinal accelerometer also allows for a *parallel estimation* of vehicle mass. We achieve this by rearranging [equation \(1\)](#), and assuming the rolling resistance to be unaffected by road grade ( $\mu \cos(\beta) \cong \mu$ ). This leads to

$$m(\dot{v}_x + g(\mu + \sin \beta)) = \frac{T_w}{r_w} - \frac{1}{2} \rho C_d A_f v_x^2 \quad (14)$$

$$m(a_x + \mu g) = \frac{T_w}{r_w} - \frac{1}{2} \rho C_d A_f v_x^2 \quad (15)$$

$$m = \frac{\Phi}{a_x + \mu g} \quad (16)$$

where  $\Phi$  is the right hand side of [equation \(15\)](#). Noting that situations may arise when the numerator and denominator of [equation \(16\)](#) are both zero, it is necessary to introduce a method that updates mass to account for this.

Mass estimates are updated using a single variable recursive least squares algorithm. In this study, we employ a formulation of the RLS algorithm for a single forgetting factor as described by Johnson [\[18\]](#). Here, we improve and extend the RLS algorithm to consider cases where the right hand side of [equation \(16\)](#) is nearly indeterminate. A potential approach to resolve this concern is to introduce a variable forgetting factor. However, since vehicle mass is considered to be a stationary parameter over the course of a trip, the forgetting factor should already be close to unity. A variable forgetting factor also



affects RLS convergence. Instead, the estimate  $\hat{m}$  is updated depending on  $\rho_{a_x, \phi}$ , which is the correlation between measured acceleration and measured torque.

$$\hat{m}_k = \hat{m}_{k-1} + K_k * \rho_{a_x, \phi} * (\phi_k - \hat{m}_k(a_{x,k} + \mu g)) \quad (17)$$

$K_k$  is the gain calculated within the RLS algorithm, and is multiplied by the correlation between the numerator and denominator of [equation \(17\)](#). This ensures that when they are poorly correlated, the estimator reduces the contribution to the new mass estimate. Therefore, the proposed PMG method *independently estimates* both mass and grade assuming availability of an on-board accelerometer.

## EXPERIMENTAL MEASUREMENTS AND DATA PROCESSING

Experimental data was collected using a 2011-MY Ford Taurus SHO at Ford's Dearborn Development Center. [Figure 2](#) summarizes the data flow amongst the sensors, data acquisition system, and computer for the evaluation of mass and grade estimation methods.

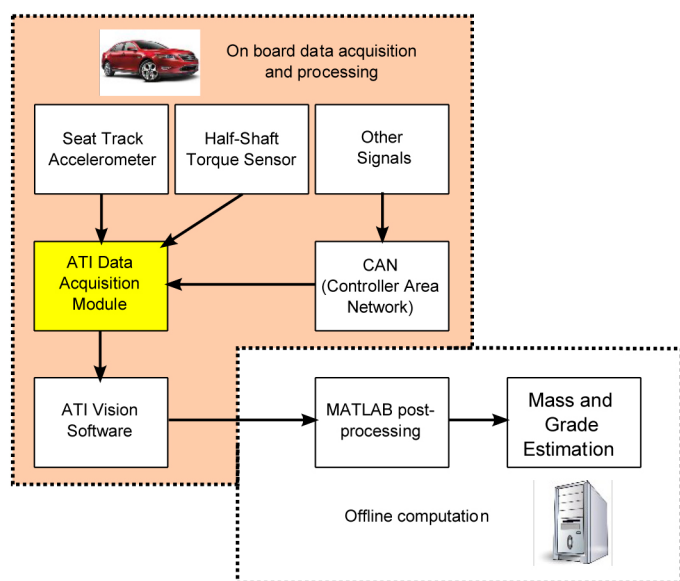


Figure 2. Flow of data during experiment and post processing

The vehicle was equipped with strain gauge sensors to measure torque on the driver's side and passenger's side half-shafts. These measurements were used instead of calculating wheel torque from the engine torque as estimated by the Powertrain Control Module (PCM). This prevented uncertainties in torque calculations from affecting the evaluation of the selected estimation methods. Implementation in a production vehicle may require use of PCM-calculated engine torque or another appropriate torque measurement along the driveline. Mass and grade estimation accuracy would then also depend on the fidelity of the drivetrain model and accuracy of the calculated engine output torque. The average of the two undriven wheel speeds was used to determine

vehicle velocity and longitudinal acceleration. Longitudinal acceleration as used in the RLS and EKF estimators was calculated by differentiating the average of the undriven wheel speeds. A seat track-mounted longitudinal accelerometer was used to carry out PMG estimation. The vehicle's drag and rolling resistance coefficients were estimated by using the vehicle model in [equation \(2\)](#) and coast-down test data sets. Data was collected either directly from sensors or via the Controller Area Network (CAN) to an ATI data acquisition module, and stored on a laptop computer running ATI's Vision software before being exported to MATLAB. Offline computation and data processing methods were acausal to accurately replicate the constraints of real-time estimation. Sensor sampling rates varied; therefore, all data was resampled at 30 Hz which represented the slowest sampling rate of the system.

The vehicle mass was measured before and after the test procedure, and the small difference attributed to fuel use. The average of the initial and final mass measurements, 2309 kg, is used here as the true vehicle mass. Several launch events were performed on flat ground in which the vehicle started at rest and accelerated under constant accelerator input. Flat ground testing took place in both directions on the low- and high-speed straightaways at Ford's Dearborn Development Center in order to account for environmental factors such as wind. Launches were performed under both fixed gear and variable gear, and with a variety accelerator positions. Grade estimation performance was evaluated by launching on flat ground and ascending an 11.8% grade hill, reflecting a grade change that may be encountered in locations with rich topography.

Although road grade is typically given as a percentage, it indicates the absolute grade and not a grade error. For a slope making angle  $\beta$  radians with the horizontal plane, the grade is defined as  $100(\tan^{-1} \beta)$ , or the change in elevation as a percentage of horizontal distance traveled. This convention is used in the following presentation of experimental results.

## EVALUATION AND COMPARISON OF ESTIMATION METHODS

In this section, the two simultaneous mass and grade estimation methods (RLS and EKF) are evaluated using vehicle launches at constant accelerator pedal angle and a mass selection scheme is proposed to identify a converged mass estimate. Then, all methods (RLS, EKF, DGO, and PMG) are evaluated on flat ground and on a sloped hill.

The estimators were implemented on data if the following instantaneous conditions were met which represent satisfaction of the assumptions in the modeling of longitudinal dynamics by [equation \(1\)](#).

1.  $v > v_{min}$ : Vehicle speed above minimum threshold.
2. Brake not applied.
3. Gear shift not in progress.
4.  $\delta < \delta_{max}$ : Steering angle below a maximum threshold.

While any of the above conditions were not met, estimation was paused and restarted upon satisfaction of the conditions. The on-line mass and grade estimates determined at the time of paused estimation were maintained during this interval.

Since grade estimation in the PMG method does not require calculation of tractive forces, the four conditions for estimator implementation were not applied. Because the data sets included only horizontal line driving, it was assumed that vehicle roll, yaw, and pitch motions were negligible. However, in real-world application, an additional condition would be necessary for implementation of longitudinal dynamics-based estimation that would omit data with excessive vehicle rotations about the center of gravity.

### Evaluation of Simultaneous Mass and Grade Estimation Methods

The two simultaneous mass and grade methods, RLS and EKF, are evaluated. RLS forgetting factors are 0.995 and 0.95 for mass and grade respectively, while the covariance matrix is initialized as a  $2 \times 2$  diagonal matrix with diagonal elements (0.001, 10). For the EKF method, the initialized process noise covariance matrix  $Q$  is set as a  $3 \times 3$  diagonal matrix with entries ( $10^{-1}$ ,  $10^{-5}$ , 25) and  $R$ , the observation noise covariance is set as 100. The first entry in  $Q$  is to account for discrepancies between measured and modeled velocity due to modelling assumptions and has units  $(\text{m/s})^2$ . The second and third entries are the process variances for  $\theta_1$  and  $\theta_2$ , respectively. Since  $\theta_1$  the inverse of mass in kg, its process noise is small in magnitude. The observation noise variance  $R$  implies a standard deviation of 10 m/s in measured velocity. This relatively large value is to ensure measurement noise has a negligible effect on mass and grade estimates. These values were tuned to allow similar performance in terms of settling time and accuracy, as well as similar speed in tracking the grade change from 0% to 11.8%.

### Estimation on Flat Ground

The simultaneous estimation methods are initially compared for ability to estimate a constant mass and grade when the vehicle is launched from rest. No constraints are placed on gear shifting, but estimation is paused during shifts. A representative velocity profile for this test is shown in [Figure 3](#).

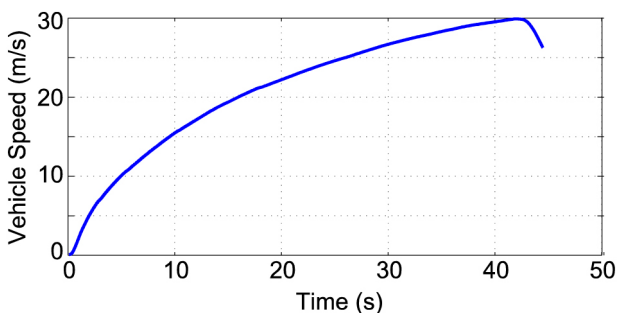


Figure 3. Velocity profile for results shown in [Figures 4, 5, 6, and 7](#)

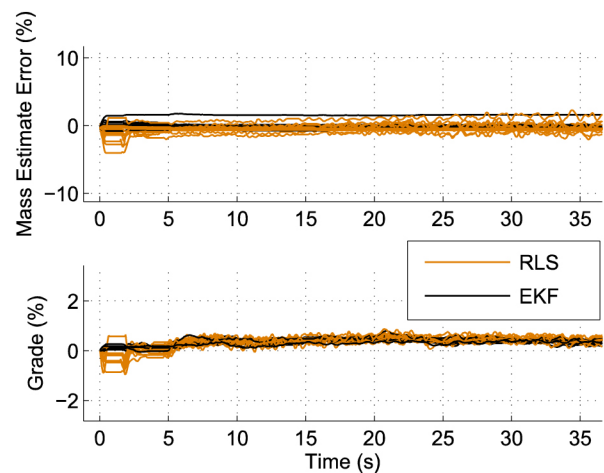


Figure 4. Simultaneous (a) mass and (b) grade estimates with accurate initial guesses. 10 data sets are plotted.

[Figure 4](#) presents mass estimate error and absolute road grade plotted as a function of time. The results from 10 data sets of near-identical launch events are shown. Accurate initial guesses for vehicle mass and road grade were provided and the two parameters are tracked well by both the RLS and EKF estimators. A scatterplot summarizing the results after 25 seconds is shown in [Figure 5](#). Note that the mass estimates are within 30 kg of the actual mass (2309 kg), which is denoted by the vertical dashed line. This corresponds to a mass error of less than 1.5%. The grade estimates after 25 seconds are between 0.2% and 0.6% (0.1 and 0.4 deg). Keeping in mind that the estimators were initialized with *correct* values of mass and grade, the errors of the estimates from actual values are likely due to acquired data which are not perfectly modeled by [equation \(1\)](#) (e.g., small tire slip or imperfectly known tire radius).

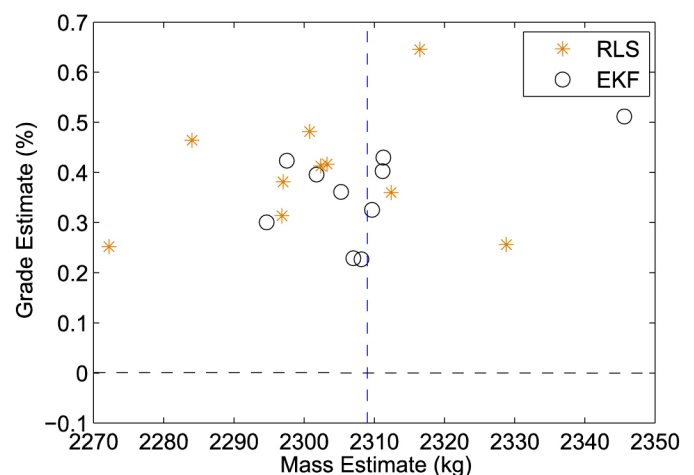


Figure 5. Scatterplot showing mass and grade estimates from [Figure 4](#) at 25 seconds

The results presented in [Figures 4](#) and [5](#) represent straightforward simultaneous mass and grade estimation for tracking constant parameters, and demonstrate that accurate initial guesses lead to fairly accurate predictions. Yet, performance under poor initial estimates must necessarily be

considered. Such a situation represents ordinary vehicle loading condition changes from trip to trip. Figure 6 shows estimation results using the same 10 test data sets as before, but with initial guesses of 1800 kg and  $-2$  deg respectively. This corresponds to an initial error of  $-22\%$  in mass and a grade error of  $-3.5\%$ .

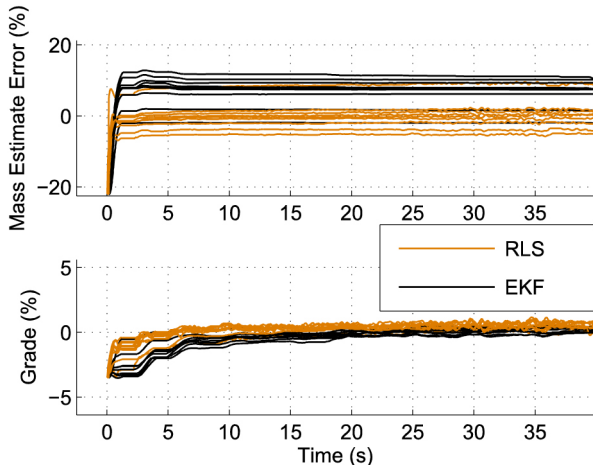


Figure 6. Simultaneous (a) mass and (b) grade estimates with inaccurate initial guesses. 10 data sets are plotted

The mass estimates adjust very quickly, then remain fairly constant during the remainder of the test. Meanwhile, grade estimates slowly adjust to the true value. This is due to mass given a much lower covariance in the EKF and RLS algorithm initializations, reflecting the fact that it is a constant parameter. Figure 7 presents the mass and grade estimates after 25 seconds.

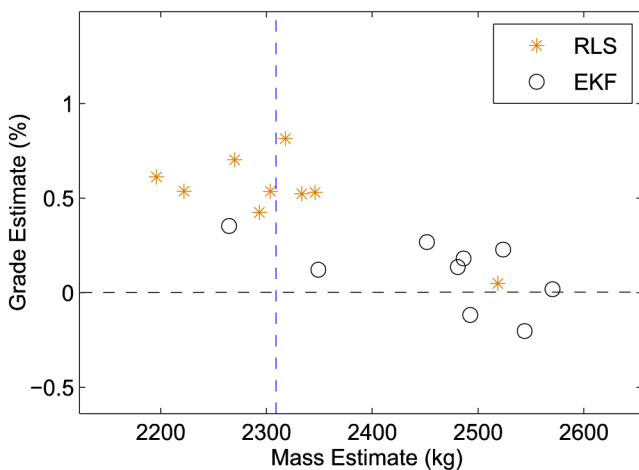


Figure 7. Scatterplot summarizing mass and grade estimates from Figure 6 at 25 seconds

With poor initial guesses, the variation in mass estimates as seen in Figure 7 is much greater than found in Figure 5 for accurately initialized parameters. Figure 7 shows that after 25 seconds, settled estimates range from 2200 to 2600 kg, with RLS providing a better mass estimate but a poorer grade estimate than the EKF, potentially due to estimator tuning of the forgetting factors. Sensitivity to initial guesses is a

significant shortcoming to simultaneous mass and grade estimation methods. If vehicle loads are changed significantly between trips, neither RLS nor EKF can quickly estimate the new loads. Thus, if rapid parameter convergence is required by vehicle control systems, *simultaneous* mass and grade estimation cannot be reliably employed unless initial guesses are accurate. If rapid convergence is not essential, simultaneous estimation may be employed given sufficient excitation of vehicle longitudinal dynamics, as demonstrated in the following section.

### Mass Selection Method

Simultaneous mass and grade estimation methods are able to accurately track constant parameters when provided with good initial guesses. However, as seen in Figure 6, simultaneous estimation during a single acceleration event produces settled mass estimates that may have significant error.

A longer data set may allow for more accurate settled estimates, but it is necessary to decide when an estimate may be considered accurate. A mass estimate that remains constant for a certain period of time is not necessarily an indication of accuracy and in practice no "true value" of mass may be pre-determined for evaluation of estimation convergence and accuracy. To address this issue, this research proposes a mass selection method and validates its successful employment on a data set spanning a greater length of time.

The proposed method relies on the convergence of several EKF estimation algorithms running in parallel, but with different initializations. Every five seconds, a new EKF algorithm is begun with re-initialized covariance. Mass and grade are initialized to the most recent estimate. This permits a rapid mass estimate adjustment, similar to the behavior observed at the start of each test run in Figure 6.

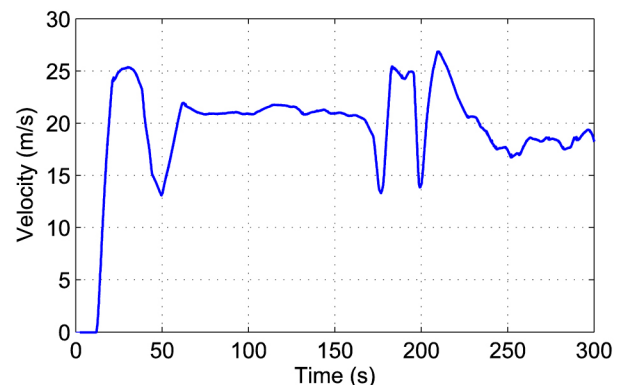


Figure 8. Velocity profile #2 with acceleration and braking events

Figure 8 shows a velocity profile for a data set collected on flat ground. The profile consists of acceleration and deceleration events, including numerous shifts of gear, as well as periods of constant speed travel. Figure 9 shows the proposed method implemented with the velocity profile from Figure 8. Initial mass and grade guesses are poor. The initial guess for vehicle mass has 55% error and the initial guess for road grade is  $-5\%$ .

Each new instance of the EKF algorithm with reset covariance is denoted by a circle and runs concurrently with all previous estimators. The concurrent estimates do not, however, interact with exception that each new iteration begins with initial values representing the instantaneous estimates of mass and grade of the algorithm iteration begun just prior.

In comparing Figures 8 and 9, it is seen that each acceleration and deceleration event coincides with a notable shift in mass estimate. This is due to estimation being paused during the braking deceleration and restarted at a significantly different velocity, resulting in a linearization about a new operating point and requiring the EKF to adjust the mass estimate accordingly. This behavior is particularly evident around 50, 175, and 200 seconds. In the absence of these dramatic excitations, the mass estimates remains relatively constant, as seen in Figure 9 between 60 s and 130 s. The five concurrent EKF algorithms all have static mass estimates during this period, but with errors ranging from 2% to 10%. At 175 s, during a sharp acceleration event, the mass estimates in Figure 9(a) converge to almost the same value. Once this occurs, the difference between true mass and estimated mass is less than 4% for all estimators. Convergence of estimators with different initial conditions to the same vehicle mass is an indication that the estimated value of mass satisfies equation (1) in such a way that all EKF algorithms update identically thereafter. Thus, the EKF algorithm implementations should agree to any further changes in mass and grade estimates, which is apparent in Figure 9(a).

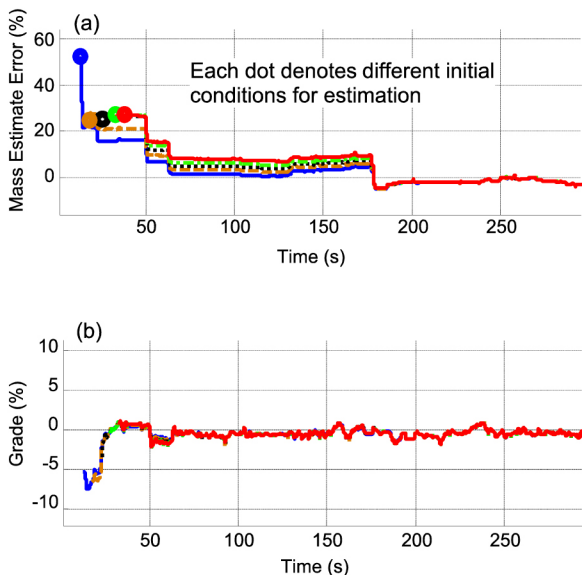


Figure 9. Parallel estimation using EKF approach for velocity profile shown in Figure 8. (a) Mass estimates with unique dots representing different initial conditions. (b) Grade estimates.

Estimates are considered to have converged to the same value if the following condition is met:

$$c_v(\hat{m}) = \frac{\sigma(\hat{m})}{\mu(\hat{m})} < \tilde{c}_v \quad (18)$$

$c_v$  is the coefficient of variation, a dimensionless number defined as the sample variation divided by the sample mean.  $\tilde{c}_v$  is a tunable threshold for converged mass estimates. Table 1 summarizes estimation results for velocity profile #2 (Figure 8) with  $\tilde{c}_v = 0.01$  under a variety of initial guesses. Note that the convergence of estimates tends to coincide with excitation of vehicle longitudinal dynamics.

Table 1. Time until mass estimate convergence under different initial guesses for mass and grade.

Initial guess (Mass error %, Grade %)	Time to $c_v(\hat{m}) < 0.01$ (seconds)
(-55%, 5%)	177
(55%, 5%)	184
(-22%, 3.5%)	177
(22%, 3.5%)	131
(-22%, -3.5%)	131
(22%, -3.5%)	131

Mass estimates eventually converge to the true value after several acceleration events shown in Figure 8. If initial parameter guesses are poor, it is necessary to provide significant excitation of longitudinal dynamics before the coefficient of variation is below the threshold to consider the mass estimate to be accurate. Highway driving at constant speeds may not provide numerous events of notable excitation to the longitudinal dynamics; in this case, the proposed method for mass and grade estimation may not be suitable. Further assessment of the method is needed to form more decisive conclusions to its efficacy amongst various driving scenarios and vehicle loading conditions.

## Evaluation of All Mass and Grade Estimation Methods

### Estimation on 11.8% Grade Hill

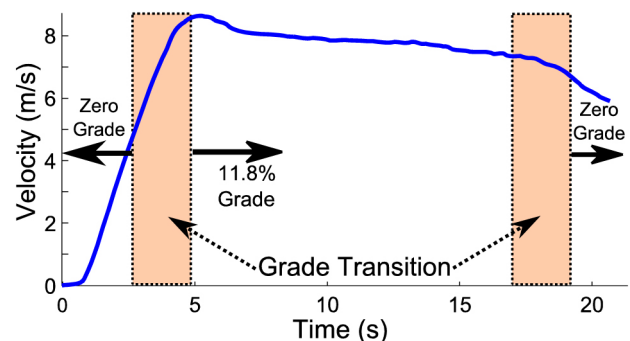


Figure 10. Velocity profile #3 for results shown in Figures 11, 12, 13. Shaded regions indicates approximate time of transition between zero grade and 11.8% grade as estimated by kinematic approach.



This section assesses the performance of the simultaneous estimation methods (RLS and EKF) in addition to the dynamic grade observer (DGO) and parallel mass and grade (PMG) approaches. The driving profiles began on a zero-grade surface and then climbed a constant grade of 11.8%. The velocity profile for the test results presented in this section is shown in Figure 10.

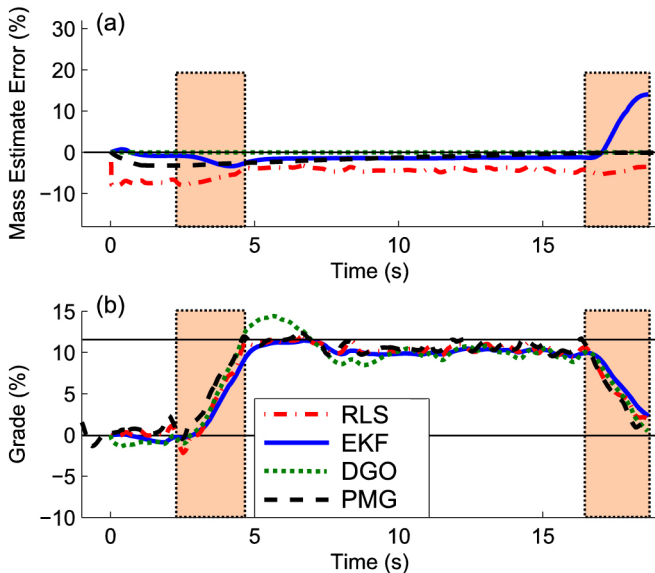


Figure 11. (a) Mass and (b) grade estimation on 0% grade and 11.8% grade. Thin solid lines indicate actual mass and the two known grades. Shaded region indicates transition between 0% and 11.8% grade. All methods are provided accurate initial guesses where needed.

Figure 11 shows that all methods are able to track changes in grade to a similar degree of accuracy when given correct initial guesses. Mass estimates stay within 5% of the true mass, indicating that the simultaneous mass and grade methods (RLS and EKF) are able to effectively separate the dynamic effects of mass and grade while the parallel estimation of the PMG approach can likewise follow dynamically changing grades and maintain good estimates of mass throughout the transitions. (Recall that the DGO approach does not estimate mass and therefore its accurate initial guess remains constant).

However, as was the case for constant, zero-grade data sets, the simultaneous estimation methods are less accurate in terms of mass prediction when initial guesses are poor for the present data set involving dynamic grade change. This is illustrated in Figures 12 and 13, which present results for two different inaccurate initial guesses: -22% and 3.5% errors for mass and grade, respectively in Figure 12 and 72% and 3.5% errors for mass and grade, respectively in Figure 14. In these events, the accuracy of mass predictions by RLS and EKF degrades substantially due to poor initial guesses, in spite of the significant excitation provided to the vehicle by way of the transient grade profile (and hence longitudinal dynamics excitations). This is a significant shortcoming of the RLS and EKF methods.

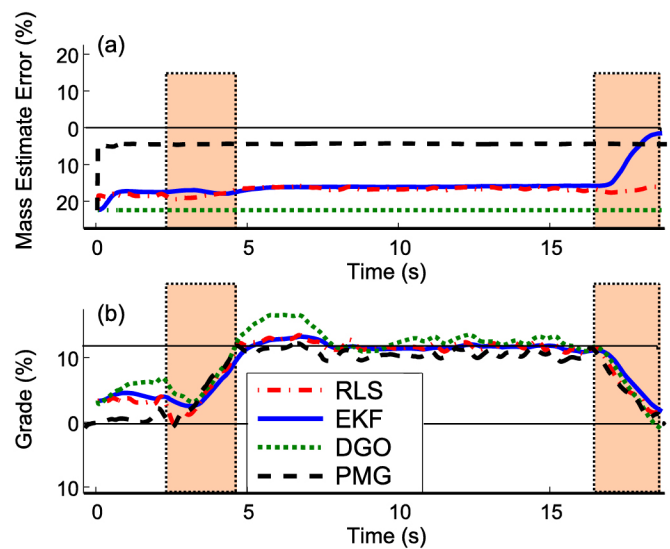


Figure 12. (a) Mass and (b) grade estimation on 0% grade and 11.8% grade. Thin solid lines indicate actual mass and the two known grades. Shaded region indicates transition between 0% and 11.8% grade. The initial mass guess had an error of -22%, and the initial grade estimate was 3.5%.

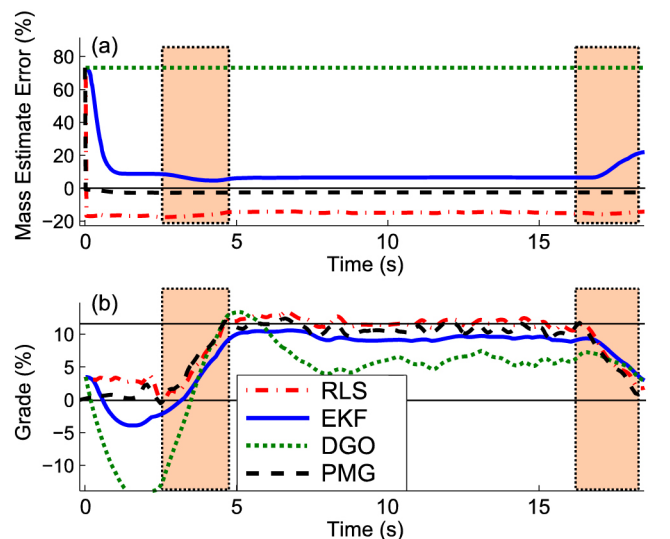


Figure 13. (a) Mass and (b) grade estimation on 0% grade and 11.8% grade. Thin solid lines indicate actual mass and the two known grades. Shaded region indicates transition between 0% and 11.8% grade. The initial mass guess had an error of 72%, and the initial grade estimate was 3.5%.

Figures 12 and 13 illustrate the sensitivity of the DGO method to an incorrect assumption of vehicle mass. If the mass parameter is wrong, it is compensated for by the grade estimate to satisfy equation (1).

In great contrast to RLS, EKF, and DGO methods, the PMG estimator developed in this work is shown in Figures 12 and 13 to yield accurate estimates of mass and grade regardless of the initialized values of mass. Mass estimates from the PMG method converge rapidly to true vehicle mass and are unaffected by changing grade. Furthermore, the ability of the accelerometer to capture the effects of longitudinal acceleration

as well as road grade means no differentiation of vehicle velocity is necessary, which is a data processing benefit as relates to real-time application of the approach. Given an on-board longitudinal accelerometer, the accurate and robust estimation of the PMG method indicates that mass-compensated controls may be successfully realized. One of the most significant advantages to grade estimation using this approach is that it is not subject to the on/off constraints on longitudinal dynamics that apply to all other methods, since kinematic data is available regardless of vehicle velocity or the state of the transmission or friction brakes. The trend towards more advanced and comprehensive vehicle sensor sets suggests that availability of such an accelerometer is more likely in the future and supports the viability of the PMG estimation approach.

## SUMMARY AND CONCLUSIONS

Knowledge of vehicle inertial loads is crucial to the success of many automotive controllers for performance, economy, and passenger safety. In order to reduce computational effort and inconsistency amongst controllers, it is beneficial to provide unified and accurate estimation of inertial loads governed by the vehicle mass and road grade. Nevertheless, integration of real-time vehicle mass and grade estimation to vehicle technologies is not widely observed in today's vehicles.

To help explain the reasons for the lack of adoption of real-time vehicle inertial parameter estimation and to propose alternative methods and improvements to existing approaches, this paper evaluates several vehicle mass and road grade estimation methods present in the literature. The accuracy and reliability of these methods are evaluated using numerous experimental data sets. Simultaneous mass and grade estimation methods are found to be effective at tracking a constant mass and changing grade within 5% and 2% respectively, if accurate initial guesses are provided. When these initial guesses are inaccurate, estimates may converge to values that retain significant residual error.

In order to address this shortcoming and identify conditions under which a mass estimate may be considered accurate, a mass selection algorithm using the coefficient of variation of multiple estimator initializations is proposed. It is implemented with the EKF method and the results show good promise for accurate mass estimation in spite of poor estimation parameter initialization. Further study is required to form more decisive conclusions regarding the utility of the approach to a wider range of driving scenarios.

The most reliable and accurate estimation results are shown to be obtained by parallel mass and grade (PMG) estimation using a longitudinal accelerometer as developed in this research. The kinematic information provided by the accelerometer decomposes the influence of road grade on longitudinal dynamics, enabling reliable mass and grade estimates with rapid convergence in spite of poor initial

guesses. Given automotive trends towards more advanced sensor technologies and present increasing adoption of high-quality accelerometers in vehicles to date for use with other control systems, the PMG estimation approach demonstrates the greatest potential for successful real-time mass and grade estimation to advance the performance, economy, and reliability of future vehicle controls.

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